



Master Thesis

**COMPUTABLE GENERAL EQUILIBRIUM MODELS  
APPLIED TO ENERGY POLICY EVALUATIONS**

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# 1. INTRODUCTION

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This document summarizes the research carried out within the Master Program of Electrical Energy Systems at the Universidad Pontificia Comillas of Madrid by the student Renato Dias Bleasby Rodrigues. The Master work addresses the problem of developing a model suitable to assess the consequences of specific electricity policies that requires simultaneously the evaluation of macroeconomic repercussions and the assessment of demand and production technological displacements at the electricity sector. Demand response and energy efficiency encouragement programs can be underlined as examples of such policies.

The development of this work has two main stages, divided in three parts:

**Stage 1: Literature review:**

1. Research work I: State of the art of Computable General Equilibrium models applied to energy policy evaluations.

**Stage 2: Personal Contributions:**

2. Research work II: Development of a Computable General Equilibrium (CGE) model for Spain and an application to demand response programs evaluation;
3. Research work III: An electricity extension for CGE models.

## **Stage 1: Literature review:**

### **1. Research work I: State of the art of Computable General Equilibrium models applied to energy policy evaluations**

The main objective in the first part of this work was to review and analyze the most used macroeconomic models in the literature for energy-economy-environmental (E3) assessments and to assess their suitability for representing policies affecting the electricity sector. Hence, as a result of this work, the paper entitled “*State of the art of Computable General Equilibrium models applied to energy policy evaluations*” was presented by the author as his first year research work, requisite of the Master Program of Electrical Energy Systems (see section 2 for the paper).

The main conclusions of this work are the following:

- The alternative partial equilibrium approach is not capable of dealing entirely with indirect substitution, income and rebound effects necessary to evaluate a series of policy consequences. Therefore, CGE models can represent a meaningful auxiliary assessment tool for ex-ante simulation of policy interferences.
- Unfortunately this advantage does not come without trade-offs:
  - CGE models are not designed, in principle, to predict the behavior of variables, and as such will never provide a direct empirically verifiable result. Their relative price structure configures it as a mostly allocative assessment tool, suitable for comparative static evaluations and

- methodical formulation of cause-effect relationships between economic variables and their repercussions.
- Economic assumptions traditionally found in CGE models like homogeneous capital, flexible prices, steady-state growth path and perfect-competition markets should be evaluated case by case according the application in mind and the market to be modeled.
  - The high nonlinear structure of the price-demand relationships in these models implies a series of mathematical caveats that should be taken into account, such as the existence of multiple equilibriums, instability and even nonexistence of equilibrium that arises under more realistic economic assumptions.
  - Some of the above cited limitations could affect either general or partial equilibrium models. Nevertheless, a robust Computable General Equilibrium formulation should:
    - Make explicit the set of microeconomics assumptions accordingly to its specific application;
    - Make explicit and justify the macroeconomic closure rules and, for the appropriate case, the ‘dynamical’ accumulation process chosen for the economic simulation;
    - Evaluate the accuracy and reliability of the technological descriptions embedded within the model; evaluate the existence and stability of a solution, especially in multiple equilibriums situations, mainly through the use of sensibility analysis;
    - And finally it is necessary to always keep in mind the limitations embedded in the endeavor of representing complex human and physical relations through the utilization of simplified mathematical models.

### **Outcome obtained from the research work I:**

Besides its utilization as the Master first research work, the contents of this literature review were also used as basis for the writing of the book chapter entitled “Energy-Economic-Environmental models: a survey”<sup>1</sup>, to be published at an E3 Climate Change Handbook edited by Edward Elgar Publishing Ltd.

## **Stage 2: Personal Contributions:**

### **2. Research work II: A Computable General Equilibrium model for Spain and an application to demand response programs evaluation**

Based on the obtained results from the research work I (literature review), the main objective of this second part of the master work was to formulate a CGE model specific to Spain capable to be applied for demand response evaluation. The model and its respective application are detailed in the paper entitled “*Production and Emissions*”

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<sup>1</sup> Book chapter authors: Renato Rodrigues (IIT, first author); A.G. Gómez-Plana (Public University of Navarre) and M. González-Eguino (Basque Center for Climate Change).

*Impact of Household Electricity Demand Response: A CGE Assessment for Spain*” (Section 3).

Specifically for this purpose, a CGE model of neoclassical formulation, static, which models the relations of a country (Spain) with two outer regions (Europe and Rest of World), with the presence of two production factors (capital and labor), two institutions (government and representative household), twelve equivalent taxes based on Spanish system and 68 productive sectors was developed.

Using previous results provided by the bottom-up client and electricity model developed within the scope of the project CENIT-GAD (Conchado and Linares, 2009a, 2009b)<sup>2</sup>; the potential displacement and downsizing of electricity demand due to household participation on demand response programs was applied to the CGE model in order to evaluate the indirect effects on production and emissions levels of other sectors on the Spanish economy.

The main conclusions of this work are the following:

- The evaluation of the expected effects of demand response policies is important in order to understand the different economic incentives promoted by the price signals. Moreover, these economic incentives do not concern only the electricity sector but also other economic sectors as a consequence of the matrix of inputs and outputs of the industries and therefore a general equilibrium evaluation can be justified in this problem.
- Firstly, the paper shows that there are two kinds of impacts that need to be taken in account in the evaluation of the demand response policies: the direct and indirect effects:
  - On one hand, the direct impacts address the expected changes in costs of the others industries induced by the changes in electricity production levels and prices. Concerning this impact it is expected that sectors with larger cross input/output interaction with the electricity generation experience a larger effect, as was underlined, in particular the cases of fuel industries (as natural gas and coal).
  - On the other hand, the indirect effect appears from the consequences that electricity prices changes would have over other players’ revenues and profits. The decrease on electricity costs may decrease other sectors costs as well as increase their production, consequently increasing their own electricity consumption and presenting substantial rebound effects.
- Therefore, the alterations of demand levels promoted by demand response programs could bring substantial shifts in the production structure, costs, and level of emissions of non-electricity sectors leading to a reduction of the beneficial electricity sector effects on emissions reduction.

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<sup>2</sup> Conchado, A., & Linares, P. (2009a). Evaluación del impacto de precios horarios variables (RTP) en el sistema de generación eléctrica. AEEE - Asociación Española para la Economía Energética. WP-2009-018.

Conchado, A., & Linares, P. (2009b). Gestión activa de la demanda eléctrica: simulación de la respuesta de los consumidores domésticos a señales horarias de precio. AEEE - Asociación Española para la Economía Energética. WP-2009-020.

### **Research work II outcome:**

This paper was presented at the Fifth Congress of Spanish Association for Energy Economics (AEEE), Vigo, January, 2010, and was awarded the prize for the best paper presented by a young researcher (under 35 years) at the congress.

### **3. Research work III: An electricity extension for CGE models**

The research work II accomplished one of the steps in the analysis of demand response policy impacts; however it also underlined a series of limitations implied by its modeling choice, which should be addressed at future research.

The model used in the previous research work refers to a sequential formulation where the first stage results from a bottom-up engineering electricity model are applied into a second stage pure CGE model. At the chosen framework, the indirect effects of demand response policies at the CGE level are only evaluated at non-electricity sectors, because the electricity variables are considered exogenously determined by the bottom-up model. Moreover, increasing the electricity demand responsiveness of the consumers causes a change in their consumption profile, consequently different electricity production technologies would be diversely affected by such programs. The rigidity of the traditional CGE technological formulation, based exclusively on economic production functions and substitutions elasticities, is incapable to address endogenously these demand displacements through time within a year, limiting its potential conclusions in such a policy assessment.

The research work III objective represents the first step to overcome this limitation and correctly reproduce the load block behavior of the electricity production choices inside a CGE framework. Its contents aim to provide a consistent solution to the data compatibility issue between bottom-up engineering data and top-down CGE models, i.e. it aspires to represent both supply technological richness and demand time heterogeneous electricity behavior in a way that is consistent with a CGE social accounting framework.. The paper derived from this work is entitled "*Improving the representation of the electricity sector in computable general equilibrium models*" (Section 4).

The main conclusions of this work are the following:

- The paper presents a detailed procedure in order to integrate technological information and, for the first time in the literature, load block differentiation on demand and electricity production levels improving the sector representation on CGE models and widening their set of possible applications.
- Representing the time dependability of electricity production enable such models to analyze different electricity elasticity behaviors (mostly important in issues like the formulation of demand response programs costs and benefits) or tariff design problems where assembling correctly energy-only and access-tariffs require time discrimination. In particular, the correct representation of peaking demand should be taken as crucial in E3 models because they involve

considerable economic, environmental and technical inefficiencies due to very low utilization factors.

- The procedure presented in the paper makes use of a multi-criteria goal programming decision problem, illustrated with Spanish data, easily reproducible to other markets and different technological disaggregation. Its objective is to minimize the deviations of top-down data structures originated from the inclusion of bottom-up technological detail and load block information, while still respecting the zero profit and market clearing conditions embedded in the National Accountable frameworks.
- Nevertheless, two limitations can be pointed out in the calibration process used:
  - First of all, the calibration process presented on this paper does not discriminate the deviations contributions on the objective function. Therefore, concentrating all deviations in a specific load block or distributing them through all load blocks can represent multiple feasible solutions. This clearly represents an unwanted solution that should be addressed in the calibration problem.
  - Secondly, and more significantly, the calibration procedure clearly pointed out to a disproportionate disparity between the share of combustibles use on bottom-up and top-down data estimations. The discrepancy between the estimated fuel weights in thermal generation expenses caused repercussions on the electricity generation factors deviations and, as a result, increased the deviation of T&D factors and intermediate input shares estimated.
- Additional studies are being carried out to deal with the two issues identified on the calibration process. An alternative deviation weight is being implemented to avoid the concentration of errors in specific decision variables, meanwhile the global calibration result is maintained. By the other side, improvements in data quality are being applied in conjunction to enhancements on the models equations and in the representation of the disparity on shares of fixed and variable costs payments for each load block in order to better represent the different revenues destinations under different price levels. This last adaptation is of paramount importance for correcting the higher deviations encountered in the paper results because while the variable costs are always related to the amount of electricity produced in each period, most of the fixed costs should be paid by the profits acquired at peak periods due their higher prices and consequently higher revenue on the period. This important feature was not taken into account in the preliminary version of this paper and offers a substantial alternative to improve the compatibility of Top-down and Bottom-up data in the estimations..

### **Research work III outcome:**

This paper was presented at the 33rd IAEE International Conference, Rio de Janeiro, June, 2010.

The extensions proposed above are being implemented in the paper at this moment. Once done it, a CGE application evaluating demand response program effects on Spanish economy will be added to its contents in order to exemplify policies that can only be

correctly addressed on models capable of reproducing simultaneously the displacement and downsizing of demand and production technologies. Finally, the final contents are aimed to be sent for publication at the Energy Economics journal (JCR impact factor=2.333) were most of the related references were published in the last few years.

## **2. RESEARCH WORK I**

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### **INTRODUCTION**

This section contains the research work I entitled:

**STATE OF THE ART OF COMPUTABLE GENERAL EQUILIBRIUM MODELS  
APPLIED TO ENERGY POLICY EVALUATIONS**



# STATE OF THE ART OF COMPUTABLE GENERAL EQUILIBRIUM MODELS APPLIED TO ENERGY POLICY EVALUATIONS

Renato Rodrigues

June, 2009

## **Abstract:**

Finding tools to assist economic decisions is a laborious process. Computable general equilibrium (CGE) models have been applied as one of the alternatives to this subject since the early 1970s. This paper presents a survey of the state of the art of this type of models and its most common applications. The objective of the study is to formulate an introductory assessment on the capability and limitations of CGE models applied to evaluate alternative energy policies. The CGE mathematical and economic assumptions are presented and their applications and extensions are evaluated. The paper attest that CGE ability to endogenize market relations and deal simultaneously with substitution and income effects, which provide an advantage when compared with partial equilibrium models whereas specific economic assumptions and mathematical complications deviates its results from reality. These conclusions point to a classification of CGE as an auxiliary cause-effect assessment model instead of a prediction tool, being capable to match a desirable range of applications if carefully formulated.

**Keywords:** Computable General Equilibrium (CGE), Social Accountability matrix (SAM).

**JEL classification:** C68, D58

## **I. Introduction**

Verify alternative policy consequences can be an arduous theoretical process and even a too-complicated procedure to sustain clear answers. Empirical and theoretical models have been developed to make easier this complexity and facilitate the decision making process.

The most usual partial equilibrium approach disregards explicitly or implicitly the impact on endogenous variables not directly related to the problem. For example, the clearance on the studied market could be independent from prices and quantities demanded and supplied in other markets.

Sources of linkages across markets, like wealth effects, can be treated exogenously in partial equilibrium models. However, in order to evaluate policy interventions that affect large numbers of markets simultaneously these linkages cannot be neglected. A general equilibrium approach is then necessary.

The general equilibrium formulation considers many markets simultaneously, unlike partial equilibrium theory which considers only one market at a time. The resulting model views the economy as a closed and interrelated system in which the equilibrium values of all variables of interest must be determined simultaneously.

In energy and environmental issues, the relevance of the attributes overlooked in the partial equilibrium approach can be meaningful. The energy sectors weight in the determination of economic levels, their huge interrelation with other productive sectors, and their significant environmental influence provides evidences pointing to possible modeling advantages reachable in a general equilibrium perspective.

Computable general equilibrium (CGE) models have been applied as a tool to assist economic decisions since early 1970s. Evolving from Leontief's 1930s multi sector input-output models, one work is usually referenced as the seminal work in the CGE subject: Johansen's (1960) model of applied general equilibrium to analyze economic growth in Norway. Ever since, the CGE universe of applications expanded from fiscal issues to evaluation of commercial and environmental policies, structural adjustments, income distribution, specific-sector production strategies, etc.

This paper presents a survey of the state of the art of CGE models and its general applications. The objective of the study is to formulate an introductory assessment on the capability and limitations of CGE models applied to evaluate policy interventions.

This paper is structured as follows: Section II presents the concepts of a CGE model and its typical formulation. Section III describes the economic assumptions of the more traditional CGE models. Sections IV and V shows an introductory literature on the more common applications and possible extensions of this model. Finally, section VI analyses briefly the limitations and critiques of the CGE's assumptions, while section VII provides the conclusions drawn from the study.

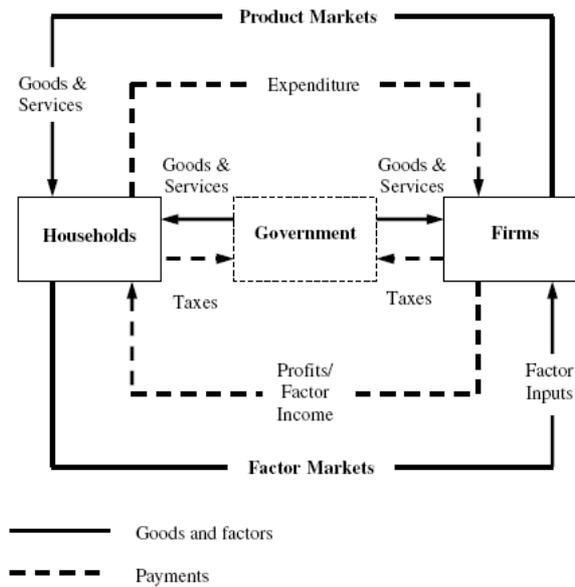
## **II. Computable General Equilibrium**

The conceptual starting point for CGE models, as presented by Sue Wing (2004), is the economy circular flow diagram, shown in Figure 1

In order to construct the economy circular flow, first the economic actors should be specified. In Figure 1 the agents are represented by the households, the government, and the firms. Second, the possible transactions between the agents should be described, specifying the existing markets in the economy. Typically, households consume goods and services from firms and in turn they rent the production factors to the firms capable of producing goods and services; the government can be treated as a transfer-only agent reallocating the final flux of goods and services through different taxes.

In the circular flow of economy product and value are conserved. The product conservation reflects that a quantity of a factor owned by a household or a commodity produced by a firm must be completely absorbed by the firms and households respectively. Conservation of value reflects the budgetary balance between the income and expenditure of the agents.

Figure 1: Circular flow of a closed economy.



Source: Sue Wing (2004).

A CGE structure is not very different from the above. Following the structure described by Robinson (1989), at first economic agents should be specified like in the circular flow of the economy. Second, one must specify behavioral rules that reflect the motivation of each agent. “For example, producers are typically assumed to maximize profits subject to technological constraints and households to maximize utility subject to income constraints.” (Robinson, 1989, p. 907). Third, the signals that influence the agents’ decision should be specified. In Walrasian’s CGEs, within Arrow-Debreu (1954) tradition, the prices are the only signals that matters to agents. Fourth, the market structure should be specified to determine the institutional structure where agents interact. For example, perfect-competition implies agents as price-takers and flexible-prices.

In traditional Walrasian’s CGEs, the concept of product conservation reflects the principle of non free disposability, implying a condition named *market clearance*. Conservation of value in the production side implies that the revenues from production should be translated into factor income to households (or payments to intermediate products and/or taxes income for government), in parallel, the value of a product must be equal to its costs of production (intermediate inputs and factors payments). Therefore, the conservation of value reflects constant returns to scale and perfectly competitive markets for produced commodities, implying a *zero profit condition*. In the household side, the conservation of value implies the full employment of owned factors, reflecting the principle of balance-budget accounting known as *income balance* or *budget constraint* (Sue Wing, 2004).

## II. a. Walrasian CGE formulation

As a basis to the following discussion on CGE capabilities and limitations, a simplified CGE model is presented in this subsection. The model presented is based on the rereading made by neoclassical economists of the Walrasian theory of markets (Walras, 1874).

Two classes of agents are represented: the households and the industrial sectors. The households are represented by a representative agent and their behavior is a consequence of the outcome obtained from a utility-maximization problem, characterized by a choice between savings ( $s_i$ ) and consumption ( $c_1, \dots, c_n$ ) of the  $n$  commodities, constrained by their income ( $m$ ) that usually emerges from their factors' ownership ( $F_1, \dots, F_f$ ).

$$\text{Max}_{c_1, \dots, c_n} \quad U(c_1, \dots, c_n) \quad \text{II. a.1}$$

$$\text{Subject to:} \quad m = \sum_{i=1}^n p_i(c_i + s_i) \quad \text{II. a.2}$$

Each  $n$ -th industrial sector produces one specific commodity ( $y_j$ ) and is composed of many similar firms that behave as profit maximizers, subject to its production technologies ( $\emptyset(\cdot)$ ), inputs' price ( $p_1, \dots, p_n$ ) and factors' prices ( $w_1, \dots, w_f$ ). The production technology depends of intermediate inputs ( $x_{1j}, \dots, x_{nj}$ ) and factors utilization ( $F_{1j}, \dots, F_{fj}$ ). Assuming perfect competition, the firms do not hold market power and accept the commodity price as given. Therefore, the sector output is equivalent to one large aggregate firm decision.

$$\text{Max}_{x_{ij}, F_{fj}} \quad \pi_j = p_j * y_j - C(p_1, \dots, p_n; w_1, \dots, w_f) \quad \text{II. a.3}$$

$$\text{Subject to:} \quad y_j = \emptyset(x_{1j}, \dots, x_{nj}; F_{1j}, \dots, F_{fj}) \quad \text{II. a.4}$$

An allocation of non-negative activity levels and income – ( $x_{ij}; y_j; c_i; s_i; F_{fj}$ ) – with non-negative product's and factor's price vectors – ( $p = (p_1, \dots, p_j)$  and  $W = (w_1, \dots, w_f)$ ) – constitute a Walrasian (competitive) equilibrium if firms maximize their profits (equations II. a.3 & II. a.4), consumers maximize their utility (equations II. a.1 & II. a.2) and the goods and factor markets clear at equilibrium (equations II. a.5 & II. a.6). In other words, the allocation of activity levels and prices constitute a competitive equilibrium if no production activity makes a positive profit (*zero profit condition*), expenditure does not exceed income (*budget constraint*) and excess supply is non-negative for all goods and factors (*market clearance*) (Mathiesen, 1985).

$$y_i = \sum_{j=1}^n x_{ij} + c_i + s_i \quad \text{II. a.5}$$

$$F_f = \sum_{j=1}^n F_{fj} \quad \text{II. a.6}$$

Any functional form could be chosen for the utility function ( $U(\cdot)$ ) and the production function ( $y_j = \emptyset(\cdot)$ ). However, a few properties are desirable in order to ensure the existence and uniqueness of the solution, as will be discussed further in this paper (see subsection VI. a).

Once the functions have been chosen, the model can be represented through a set of three types of equations: income-balance equations, zero-profit conditions and market-clearance restrictions. Finally, the set of simultaneous equations derived from the Walrasian equilibrium, added to macro-aggregate identities (expressed in subsection V. a, macroeconomic balances) corresponds to an economy-wide CGE model.

The system of equations still presents a property known as Walras’s Law, which holds that one equation is functionally dependent on the others and can be dropped. The way to go around this problem is to adopt a commodity price as a “numéraire”, calculating all the other prices as relative prices in relation to this commodity’s price.

## II. b. Social accountability matrix

The data requirement for a CGE formulation is dependent on the functions chosen to represent the utility and the production. Usually, historical substitution elasticity between products and factors obtained through econometric estimations are necessary (see section V. b).

All together, comprehensive economic data are still needed to completely specify the model. The Social Accountability Matrix (SAM) has been used for this purpose as a framework to consolidate the economic wide data, typically representing the macro-aggregates and input-output sectors information of a nation. The SAM is a ‘snapshot’ of the economy that embodies information normally included in national accounts and other sources.

One simplified example of an SAM matching the CGE model presented on subsection II. a is illustrated in Table 1.

Table 1: Social accountability matrix (SAM).

		Expenditures							
		Sectors Intermediate			Production			Final	
		Inputs			Factors			Demands	
		Sector 1	...	Sector n	Factor 1	...	Factor f	Household Consumption	Investment
Receipts	Sectors	Sector 1	1	...	2			1	3
		...	...	...	...			...	...
		Sector n	2	...	1			2	1
	Production Factors	Factor 1	3	...	1				
		Factor f	1	...	2				
	Institutions	Household Ownership				4	...	3	
	Capital account	Investment							4

Source: Own elaboration.

The SAM matrix core is the circular flow of demand – from industries to intermediary products and factors, and from institutions to consumption goods and investment –, of production – detailed typically as in traditional input-output tables – and of income – receipts for all agents according their initial ownerships. The SAM formulation incorporates the income balance restriction intrinsically.

“More technically, a SAM is a square matrix in which each account is represented by a row and a column. Each cell shows the payment from the account

of its column to the account of its row. Thus, the incomes of an account appear along its row and its expenditures along its column. The underlying principle of double-entry accounting requires that, for each account in the SAM, total revenue (row total) equals total expenditure (column total).” (Löfgren, Robinson, & Lee Harris, 2002, p. 3)

### III. Economic Assumptions

CGE are macroeconomic models consistent with micro-foundations. This means that “the demand and supply functions contained in the models are consistent with (in other words: can algebraically be derived from) the utility and profit maximization calculus which is the core of the neoclassical economic theory of consumer and producer behavior” (Bernow *et. al.*, 2002, p. 6).

The representation of economic decisions is based solely on a process of allocation of the scarce resources. The market clearance conditions not only causes that the demand for factors in all economy adapts itself to the endowment of factors available, but also promotes the complete utilization of any available resource. In other terms, the conservation of value determines that the production of goods provides the sufficient means for the producers (or for the owners of the means of production) to purchase what is produced, and hence, demand will behave as an adjustable variable, growing always when production grows. As a consequence, under full employment and markets clearing the economic equilibrium will always be obtained within the efficient productive frontier.

The mechanism that makes possible to reach such equilibrium is the principle of substitution, presented in both production and consumption sectors. This principle attests that under competitive assumptions the relative price from all the factors should be adjusted by the portfolio decisions of economic agents’ choices until the equilibrium is reached.

Partial equilibrium models also possess the ability to deal with substitution effects. Yet, this effect is only partially covered in such models. For example, if an industry utilizes an appreciable amount of a specific factor of production, an increase in the output of the industry would increase the demand for the production factor and alter its price as a partial equilibrium analysis suggests. However, at the same time the production factor affected is likely to be used in the production of substitutes for the industry's product, and as a result a change in the price of that factor will have effects on the supply of those substitutes. The competitors’ costs and supply shifts caused by the industry output decision could only be evaluated if the interactions between markets are taken into account, as occurs in a general equilibrium approach.

A clearer distinction between general and partial equilibrium analysis can be drawn by the presence of income effects. As described by the Slutsky identity<sup>3</sup>, price changes cause demand changes due to the presence of two effects: a substitution effect – that corresponds to a change in the exchange rate between two goods – and an income effect

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<sup>3</sup> The Slutsky identity is given by:  $\frac{\partial x_i(p,w)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial x_i(p,w)}{\partial w} x_j(p,w)$ , where  $h_i(p,u)$  is the Hicksian demand and  $x_i(p,w)$  is the Marshallian demand, at price level  $p$ , wealth level  $w$ , and utility level  $u$ . The first term represents the substitution effect and the second term represents the income effect.

– resulting from the changes in consumer’s purchasing power due to price changes. The income effect is important because relative price changes, besides adjusting the demand, could also modify the income of the agents, causing in turn important economic effects by shifting the supply and demand curves. For example, an increase in oranges relative prices not only could cause a fall in orange demand, but also could imply in an increase of the purchasing power of orange producers. In turn, the income increase would be spent in the purchase of products, changing the demand levels in other markets. The increase of the demand in the other markets imply a repetition of the process itself, and the resulting economic ‘equilibrium’ could present substantial differences when compared to a simplified partial approach.

The difference between partial and general equilibrium results is even clearer when dealing with energy sector issues. Due to the former described substitution and income effects, improvements in energy efficiency at the micro or plant level could not be entirely realized on the macroeconomic level. The disparity between the partial and general evaluations can under certain conditions achieve a significant magnitude limiting the effectiveness of energy efficiency policies in issues like the reduction of demand requirements or the control of greenhouse gases emissions. A relevant attention is given to this issue in the literature of the energy sector, including a specific denotation: the rebound effect (Dimitropoulos, 2007).

In order to be able to represent such effects in an economy, a general equilibrium method needs to take into account a number of macro and microeconomic assumptions. The macroeconomic representation in such models is based in macroeconomic balances postulations (see subsection V. a), and are traditionally based on neoclassical hypothesis. Furthermore, the micro-foundations employed in CGE models are also based in neoclassical microeconomics theory and incorporate possible stringent assumptions like: technologies exhibit non-increasing returns to scale, firms are price-takers, economic agents act with perfect rationality, commodities are divisible, production sets are convex, utility functions are concave, consumer utility functions are very similar across individuals, capital is homogeneous and uncertainty does not exist.

## **IV. Applications**

Several compendiums indicate the rapid development of empirical CGE applications subsequent to the works of Johansen’s (1960) and Taylor & Black (1974). Devarajan, Lewis & Robinson (1986) and Decaluwe & Martens (1987) grouped numerous models from diverse countries with different modeling objectives that use CGE modeling. Gómez-Plana (2002) offered a revision of CGE models applied to Spain; and Ginsburgh & Keyzer (1997) offered a CGE survey oriented by diverse theoretical assumptions.

Historically, Shoven & Whalley works – (1972) (1984) (1992) – in addition to the Dervis, de Melo & Robinson (1982) World Bank publication contributed to disseminate applied general equilibrium as an applied tool for policy analysis.

The advantages of solving cross-sector systems and treating problems that involved complex interrelations between economic agents and sectors collaborated to the rapidly increase universe of different applications of CGE models. Empirical works date from the Harberger (1962) paper about the incidence of taxation in a two-sector model. Scarf (1967) was a pioneer in developing algorithms to estimate the Arrow-Debreu general equilibrium with empirical data. More recently, an increase interest in developing

models capable of evaluating energy and environmental policies provided incentives to the formulation of a series of multitask large models, like the GEM-E3 model – European commission (Capros, *et al.*, 1995) –, the GREEN model – OECD (Organization for economic co-operation and development) (Burniaux *et al.*, 1992) –, and the EPPA model – MIT Emissions Predictions and Policy Analysis (Paltsev, *et al.*, 2005).

Ever since, empirical CGE models have been applied in topics covering fiscal, commercial and environmental policies like: international trade (Taylor & Von Arnim, 2007), public sector and goods (Bernow, *et al.*, 2002), agriculture planning (Wittwer, *et al.*, 2005), income distribution (Bandara, 1991), development policy (Dervis, de Melo, & Robinson, 1982), growth and structural adjustment (Robinson, *et al.*, 1993), energy efficiency and sustainability (Hanley, *et al.*, 2009), environmental issues and global warming (Böhringer, Löschel, & Rutherford, 2006).

## **V. Extensions**

CGE formulations require a series of theoretical choices embedded in their structure. To clarify this diversity and describe their alternatives, this section delineates some of the principals modeling extensions applicable to the simplified model described in subsection II. a.

Firstly, some of the principal theoretical choices in macro and microeconomic CGE modeling are briefly presented (V. a). Secondly, the technological description and the utilization of bottom-up models in CGE formulations are summarized. Finally, a set of issues related with time-dependable ‘dynamic’ CGE models is presented.

### **V. a. Economic theory**

One can argue that CGE models were first conceived as a micro-founded neoclassical equilibrium model. Nevertheless, as time passed, additional assumptions were evaluated in order to approximate its structure to macroeconomic theories and to alternative microeconomic assumptions. A preamble to these CGE economical extensions is presented in the following paragraphs.

- **Macroeconomic Balances**

Possibly, the most discussed modeling choice on CGE models refers to the determination of which macroeconomic variables will be considered exogenous or endogenous to the models. This subject is usually called closure assumptions.

As Mitra-Kahn (2008) described, the notion of economic closure dates back to Sen (1963), and was addressed by Taylor-Lysy (1979) for CGE models. The macroeconomic closure corresponds to “the simple notion that the model should consist of an equal number of equations and endogenous variables” (Thissen, 1998, p. 7).

In order to explain the importance of the macroeconomic closure issue it is necessary to express the role of macroeconomic identities in the microeconomic production determination described in subsection II. a. The economy in a macroeconomic perspective considers that the concepts of product, income and expenditure are equivalents. This equivalence is expressed in the circular flow of income (Figure 1) and

reflects both product and value conservation. As a consequence, the most basic macroeconomic identity can be represented as in the equation V.a.1.

$$\sum_{i=1}^n P_i Q_i = E = Y \quad \text{V.a.1}$$

$$\text{Aggregate Product} = \text{Aggregate Expenses} = \text{Aggregate Income}$$

The above representation is compatible with the Walrasian (competitive) equilibrium expressed in subsection II. a. Nevertheless, three additional balances are typically incorporated into the macroeconomic identity: the savings-investment balance, the government balance and the external balance.

The simplest representation of the aggregate expenditure in an economy corresponds to the decision of agents to consume goods. Still, in economic models it is common to incorporate the possibility of agents demanding specific assets which increase the productive capacity in future periods. Accordingly, the resulting aggregate expenditure decision (E) could be represented by an aggregate investment component (I) and an aggregate consumption (C) component (see equation V.a.2).

Meanwhile, if the consumers have the possibility to not consume their entire revenues in produced goods, the income aggregate (Y) could be composed by the income acquired from other agents' consumptions (C) and by a saved share component (S, aggregate savings). Consequently, the aggregate income could be represented as in equation V.a.3. Substituting the equations V.a.2 and V.a.3 in the identity expressed on equation V.a.1, one can obtain the savings-investment balance identity (equation V.a.4).

$$E = C + I \quad \text{V.a.2}$$

$$Y = C + S \quad \text{V.a.3}$$

$$I = S \quad \text{V.a.4}$$

Analogously, the inclusion of the government adds two new terms to the aggregate expenditure and aggregate income respectively: the government expenditure (G) and government income (T). Therefore, the extended capital balance including the government (T and G) could be expressed as in equation V.a.7.

$$E = C + I + G \quad \text{V.a.5}$$

$$Y = C + S + T \quad \text{V.a.6}$$

$$I + G = S + T \quad \text{V.a.7}$$

The introduction of external relations on macroeconomic balances adds a demand element to the internal production – the exports (X) – and a spending element to expenditure from the acquisition from other countries – the imports (M). The difference between the former elements (X – M) provides the resources net-transfers abroad, determining the following equations:

$$E = C + I + G + (X - M) \quad \text{V.a.8}$$

$$Y = C + S + T \quad \text{V.a.9}$$

$$I + G + X = S + T + M \quad \text{or} \quad \text{V.a.10}$$

$$(X - M) = (S - I) + (T - G)$$

As can be seen, the inclusion of the savings-investment balance, of the (current) government balance and of the external balance adds a complexity to the production level determination on the original simplified Walrasian (competitive) equilibrium presented. Implicit or explicit, nearly all CGE models up-to-date include the three balances described above. However, in order to add each pair of aggregate variables (S/I, T/G, X/M) to the new CGE ‘macroeconomic’ model, one must specify which variables would be responsible to clear each balance determining the exogenous or endogenous variables to maintain the system determined.

Typically, each balance involves a set of specific variables. In government balances, the most common variables encountered corresponds to expenditure – consumption, savings, paid transfers – and revenues – tax income, received transfers. The external balance includes the real exchange rate and the foreign savings variables – which corresponds to the current account deficit<sup>4</sup>. Finally, the savings-investment balance presents the identity between economic investment and savings.

For each balance included in the model, one must specify an endogenous variable responsible to clear the balance, while the others must be exogenously defined. This choice influences directly the model behavior and depends on the context of analysis, as pointed out by Löfgren, Robinson, & Lee Harris (2002). Besides, as well, the closure choice can be suited to represent specific country situations and approximate the representation of different macroeconomic theories, as exemplified in Table 2.

The government closure, in short-run analysis, is usually closely related to actual government practices and ‘expenditure responsibility’ of the studied country or region. The closure represented in the first column in Table 2 is suitable to countries with very restricted government actions, based in account revenues. This situation is commonly found among countries with restringing government account flexibility associated with inflation control. The expenditure – and consequently the government consumption and investment – is continually adjusted to maintain a certain and stable level of outcome, frequently related to a percentage of the GDP. Another common and very restrictive closure to government behavior is to fix its real consumption levels, thus removing the possibility of an alteration of the government consumption induced endogenously by the model and limiting the government flexibility and influence on the determination of welfare.

The external balance closure can be associated with the influence of the country in the currency exchange markets and with the existence of barriers to worldwide capital and trade relations. The example presented in Table 2, second column, represent a country open to international trade with stable factor and external transfer payments, i.e. the external adjustment is made through the balance of trade levels (exports minus imports), without national influence in the determination of the exchange rate value.

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<sup>4</sup>The current account is the sum of the balance of trade (exports minus imports of goods and services), the net-factor income from abroad (such as interest and dividends) and the net-transfer payments from abroad (such as foreign aid).

Table 2: CGE macroeconomic accounts-balance closure examples.

		Government primary deficit target	Trade Openness	Savings-driven closure (neoclassical)	Investment-driven closure
Government Balance	Government Consumption / Expenditures	Flexible residual			
	Deficit / Savings	Fixed			
	Net Government Transfers	Fixed			
	Tax income	Fixed			
External Balance	Real exchange rate		Fixed		
	Balance of trade		Endogenous adjustable variable		
	Net factor income from abroad		Fixed		
	Net transfer payments from abroad		Fixed		
Savings-Investment Balance	Investment			Endogenous adjustable variable	Exogenous function
	Household Savings			Fixed savings propensity	Flexible Savings residual
	Government Savings			Fixed, Gov. balance determination	Fixed, Gov. balance determination
	External Savings			Fixed,external balance determination	Fixed,external balance determination

Source: Own elaboration. Endogenous variables shaded in gray.

Lastly, the savings-investment balance entails one of the principal issues in economic theory: the determination of the mechanism that brings investment and savings to equilibrium, especially important in dynamic CGE (see subsection V. c). In the savings-investment case, the limitation of analyzing only current and/or conjectural situations to determine the adopted closure is more evident. An assessment of different macroeconomic theories allied with the discretion choice of the modeler is a more suitable method to determine the closure assumption related to the CGE savings-investment balance.

The classification of macroeconomic closures choices presented in this paper draws heavily from Thissen (1998). Firstly, one can describe the most frequently found savings-investment closure, usually embedded implicitly in Walrasian CGE formulation, the neoclassical closure. This closure is composed by a savings-driven assumption where “a mechanism exists such that investment is brought into equilibrium with savings at a level that guarantees full employment in the economy, or, in the words of Swan (1970), “it is assumed that whatever is saved is invested” (Thissen, 1998, p. 9). The third column, in Table 2 represents an example of a typical neoclassical savings-investment closure.

The neo-Keynesian (or forced savings) closure is described as based on the forced savings model of Kaldor (1956) (1957) and Pasinetti (1962). In this closure, the income

distribution is the mechanism responsible to determine the equality between the savings and the investment. The nominal wage rate is fixed exogenously, while the production is still determined by the supply of labor and capital; thus, the product price adjusts the equality between investment and savings by changing the income distribution of the different savings-profile agents.

The Keynesian (and Johansen) closure is expressed through the introduction of unemployment in the model. Labor is endogenous while the investment can be different from its full-employment level, allowing the government intervention through expenditure or taxes to achieve the full-employment investment level.

The structuralist closure (Taylor L. , 1990) is based on a Kalecki (1976) publication. It follows from the fact that the existence of market power causes the existence of a markup price. Consequently, in the presence of excess capacity, the real wage is different from the marginal productivity of labor and unemployment exists.

Additional closures to savings-investment balance could be adopted by the option of the direct modeling of the financial markets. As an example, savings can be represented as the supply of loanable funds while the investment is its demand, the equilibrating mechanism being done through an adjustment variable as the interest rate.

Moreover, even with CGE models being relative price models, it is possible to implement adjusting variables as money supply, wealth effects (such as Pigou effects), or Tobin's style portfolio models (with financial assets interest payments and interest clearing stock markets).

An important point should still be mentioned: the importance of the full-employment assumption in CGE applications. A full employment economy usually presents its economic level exogenously determined. As Löfgren et al. (2002) emphasizes, "if full-employment is assumed in the factor markets, these closures will yield different effects of shocks on the composition of aggregate demand, but with little or no effect on aggregate GDP" (Löfgren, Robinson, & Lee Harris, 2002, p. 16). Besides, the absence of a link between macro-variables and micro-foundations could also limit the applications of CGE models.

Furthermore, additional attention should be taken when dealing with one-period studies and welfare effects. In one-period analysis, a closure with fixed foreign saving is more standard to avoid misleading welfare effects that arise because the analysis does not capture welfare losses in later periods caused by a larger foreign debt. The same misleading results can occur in the savings-investment balance analysis, in one-period analysis, by allowing that the variation of the real investment occurs without evaluating the different capital stock availability on subsequent periods.

The choice of closure variables can change significantly the results of the model. Robinson S. (2006) and (1991) discussed the effects of macro closures; and Taylor & Von Arnim (2007), showed how different closure assumptions in a (Doha Round like) tariff discussion can produce results entirely different, and even diametrically opposite.

In order to make clear this large possible difference between the results of different closure assumptions two theoretical examples can be described.

The first one is a savings-driven model. In this model the aggregate savings are determined through a fixed savings-rate out of after-tax household income and by a government deficit. In turn, Say's Law holds in the (neo-)classical model, hence the production is determined by the factor endowments and the available technology. In this context, an import tariff reduction increases the household net-income. The funds obtained by households, through the increase of income, are distributed in their decision of what to consume and what to save. The total savings should increase; hence the investment (and future production capacity) increases as well. Therefore, the trade deficit (caused by the tariff reduction) would be compensated by capital transfers (investment), enhancing production and welfare from tariff (prices) decreases.

The second example reflects an investment-driven model, where investment remains determined by an exogenous function. A tariff reduction will not change the investment; hence the (future capacity) production will not change. Consequently, the tariff decrease will have its main impact on consumption. As the identity between investment and savings remains, the savings should not change and the households should expend the "extra-income" in consumption. The conservation of the same level of production (according to Say's Law) leads from the consumption changes to an increase in the demand for import. Therefore, the tariff reduction could cause a greater trade deficit not compensated by any changes on national production.

Consequently, different causality chains between savings and investment could imply different adjustment mechanisms of economic shocks. This fact emphasizes the necessity of the evaluation of macroeconomic closures in the model formulation, nevertheless, not only macroeconomic balances are subject to alternative model specifications. A clearer alternative in the CGE modeling can be represented by the direct modification of the micro behavior of the agents, modifying the micro-foundations of the model.

- **Microfoundations**

The microeconomic foundations of CGE models gives consistence to their formulation, nevertheless this benefit does not come without disadvantages. The mathematical assumptions applied to obtain "well behaved" functions and solutions – based on the works that followed Arrow-Debreu (1954) research in the existence and uniqueness of equilibrium – place too much emphasis on mathematical requirements, rather than on the microeconomic assumptions of the model, distancing it from a more concrete economic formulation.

Assumptions like non-increasing returns to scale, price-taker firms, perfectly rational economic agents, perfectly divisible commodities, convexity of production sets, concavity of utility functions, global consumer preferences, homogeneity of capital and labor, and nonexistence of uncertainty are almost unattainable in real world situations and should be analyzed in order to identify the more compromising hypothesis and the potentially droppable ones.

Most of the microeconomic assumptions discussion was originally developed in Industrial Organization, being originated in the analysis of partial equilibrium models. However, it is possible to extend most of the debate to the CGE analysis, especially in the substitution of traditional assumptions like constant returns to scale (CRTS) and perfect competition.

Constant returns to scale and perfect competition are undoubtedly two of the most discussed and criticized assumptions in traditional CGE formulations. The existence of market power, barriers to entry and exit the market, differentiation of products, asymmetric and sunk costs, and oligopolistic markets provide incentives to replace both former assumptions by several alternative specifications of increasing returns to scale (IRTS) and imperfect competition.

As Francois & Roland-Holst (1997) pointed out, constant returns to scale is an attractive property in terms of flexibility and parsimony, facilitating “data gathering, calibration, and interpretation of results. However, its empirical veracity is open to question. In the real world, factors are heterogeneous in quality and mobility, and changes in the level of output often involve changes in average cost, even for relatively simple production processes. While there may be uncertainty about the precise magnitude, scale economies are a fact of life and appear to be pervasive even in mature industries with diverse firm populations.” (Francois & Roland-Holst, 1997).

For these reasons, numerous works on CGE empirical modeling arose after the seminal study by Harris (1984) trying to evaluate trade liberalization under alternative specifications of returns to scale. The empirical and theoretical works confirmed that the assumptions grounded on classical trade theory can be contradicted, in magnitude and/or direction, when scale economies or diseconomies play a significant role in the adjustment process.

In parallel, assuming economies of scale involves dealing with imperfect competition models<sup>5</sup>. As Roson (2006) emphasized, “there can be imperfect competition without economies of scale. However, imperfect competition is needed to accommodate economies of scale in a market equilibrium.” (Roson, 2006, p. 7).

Instruments that are ‘equivalent’ under perfect competition can lead to distinct effects under imperfect competition (Helpman & Krugman, 1985). Therefore, strategic interaction models, like conjectural variations and game theory, can be presented as an important alternative foundation to the microeconomics structure of CGE models regardless them adding complications to issues like the existence of equilibrium (see subsection VI. a).

Therefore, constant returns to scale and perfect competition, despite the facilitation of the modeling, should be carefully addressed. Undoubtedly this confirms that alternative microeconomic elements and structures, not only the ones presented in this section, should be considered in the CGE formulation in order to approximate its formulation to reality.

## **V. b. Technological description**

Neoclassical CGE models represent the production technology through an abstract mathematical description of the production process, named product functions.

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<sup>5</sup> A recommended literature for a detailed technical and conceptual description of the issues associated with the introduction of imperfect competition in a CGE model can be represented by the works of Harris (1984), Francois & Roland-Holst (1997), Francois J. (1998) and Roson (2006), accompanied by a supporting theoretical source represented by the Industrial Organization literature.

Production functions are mathematical functions that describe the maximum physical output obtainable, at an existing technological structure, from a combination of physical inputs. Their results are always placed on the optimal product possibility frontier, through an allocation process of choice between the utilization of the most efficient production inputs.

The elasticity of substitution in such functions represents a measure of how easy it is to shift between the factor inputs. The most common assumption among neoclassical economists is a production function that holds constant elasticity of substitution (CES) because of its uncomplicated implementation. The more common functional forms can be summarized as in the Table 3<sup>6</sup>.

Table 3: Most common production functional forms.

Production function	Equation	Elasticity of substitution
CES	$Q = \sum_{i=1}^N \left( a_i \frac{1}{s} X_i^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}}$	Constant = s
Cobb-Douglas	$Q = X_1^a X_2^b$	<p><i>CES com s</i> → 1            If <math>a + b = 1</math> → <i>CRTS</i>            If <math>a + b &lt; 1</math> → <i>DRTS</i>            If <math>a + b &gt; 1</math> → <i>IRTS</i></p>
Leontief (Perfect Complements)	$Q = \left( \frac{X_1}{a}, \frac{X_2}{b} \right)$	<i>CES com s</i> → 0
Perfect Substitutes	$Q = X_1 + X_2$	<i>CES com s</i> → +∞

Source: Own elaboration.

The production function representation embodies a series of criticisms addressed by non-neoclassical economists. To classical and neoclassical economists the fact that the production function only includes information about input substitution trade-offs on the efficient production possibility frontier is not a very serious issue because of the assumption of agent rationality ('*homo economicus*'). However, heterodox economists underline the existence of a gap between the maximal efficient output and the actual produced output, which can be originated by diverse causes as uncertainty, bounded rationality and asymmetric information. This gap would cause an interior solution to the production function problem that is usually unaddressed and unreachable in a traditional CGE formulation.

Nevertheless, one of the most important economic criticisms to neoclassical production functions is the Cambridge Capital Controversy Debate. Assigning production functions to individual firms' process, despite being a simplification, is not a serious issue when compared to the problem of how to determine aggregate production functions that reflect industry, sector or economy level behavior. The aggregation of the

<sup>6</sup> For a historical reference about production functions utilization see Humphrey (1997).

heterogeneous factors contained in production functions and the measurement of the factor input capital in physical terms are very problematic issues.

Non-neoclassical economists (Joan Robinson, Piero Sraffa, Luigi Pasinetti and Pierangelo Garegnani) argued that it is impossible to conceive an abstract quantity of capital that is independent of the rates of interest and wages. This creates endogenous theoretical problems to neoclassical models because this independence is a precondition for constructing an isoquant (or production function). Thus, the isoquants cannot be constructed and its slope measured unless the prices are known beforehand. However, inconsistently, the protagonists of aggregate production functions use the slope of the isoquant to determine relative factor prices. In order to solve this problem it would be necessary the construction of a quantity-adjustable measurement of the physical capital.

Even assuming it possible to create a meaningful measure of capital services, it is still necessary for the aggregation of firms' production possibility frontiers to maintain coherence with the production frontier for the economy as a whole. As Miller (2008) describes, the Leontief's theorem – which provides the necessary and sufficient conditions for the aggregation of any twice differentiable production functions – states that “aggregation is possible if and only if the marginal rate of technical substitution of the variables in the aggregate production function are independent of the variables that are not included.” (Miller, 2008, p. 12).

Thus, in a capital-labor input situation, the Leontief's theorem requires that labor has no effect on the substitution possibilities between the capital inputs; a condition clearly invalid in the real world where the choice of capital is influenced by the quantity and quality of labor available.

Neoclassical economists (such as Paul Samuelson, Robert Solow, Frank Hahn, Christopher Bliss, among others) argued that despite these theoretical shortcomings, aggregate production functions can still be defended on instrumentalist grounds if they provide a reasonably good description of the data. This affirmation was partially confirmed by specific empirical evaluations made by Fischer, Solow, Kearn and Shaikh (see Miller, 2008, p. 14).

Even disregarding the critics of the Cambridge Controversy, the neoclassical production function still presents an additional limitation to the applicability of CGE models. As Mitra-Kahn (2008) portrayed, the agents' functional forms predetermines the share which each sector contributes to the economic activity, providing an additional rigidity to the CGE model.

“More specifically the input shares of sectors will not change if the elasticities of substitution are all equal to one, and similarly the consumption shares will not change if demands are homothetic with unit price elasticities (again Cobb-Douglas). So a CGE model could not predict, nor deal with any major structural changes like China's recent boom in manufacturing, or India's booming service outsourcing sectors. Simply because those productive parts of the economy are given a set percentage of the nation's output in the benchmark, which will not change. To make adjustments to this, one would have to post facto change these shares exogenously, but it cannot be incorporated endogenously.” (Mitra-Kahn, 2008, pp. 60-61). Consequently, CGE models should focus on small changes of a known economy structure, and should not address issues related to structural changes of an entire economy.

Limit the CGE policy analysis to applications unrelated to substantial structural changes and avoiding excessive aggregation, and/or accept the empirical justification to the theoretical aggregation problems transforms the neoclassical production functions in a possible valid simplification instrument to a CGE formulation. On the other hand, the abstraction inherent in such mathematical functions disregard aspects of physical production processes – including error, entropy or waste – and from the business processes – ignoring the role of management, of sunk cost investments and the relation between fixed overhead and variable costs. In order to surpass such last critic, the adoption of an alternative technological description related with bottom-up engineering models can be utilized in sensible sectors, as it is done in a hybrid CGE modeling approach.

- **Hybrid approach**

In a pure CGE model (i.e., an exclusive top-down economy-wide model) all sectors are represented as production functions with a substitution structure between all primary factors. To be able to incorporate specificities of certain activities, it is possible to shape the representation of a specific sector utilizing more descriptive ways (i.e., utilizing bottom-up partial equilibrium models). The model that includes the CGE structure and in unison incorporates a detailed production description of a specific sector is called a “hybrid model”.

The main objective of this approach is to better represent the sectors that possess more available and detailed data, with the intention of allowing a more refined theoretical structure to the production determination process of prices and quantities. Moreover, a more detailed representation of the interrelations inside the sector will enable the study of specific sector policies and their consequences in the entire economy, which can amplify the possible uses of a CGE model.

Regarding the bottom-up alternative, no more than a partial equilibrium approach would be necessary if the interactions between the specific studied sector and the remaining economy were negligible. However, the majority of economic sectors entail indirect effects not addressed in the partial modeling alternative.

Undoubtedly, the energy sector is one of the most representative sectors on the utilization of the hybrid approach to evaluate policy issues (see IAEE special issue, Yatchew, 2006). As Böhringer and Löschel described, “energy policies do not only cause direct adjustments on energy markets but produce indirect spillovers to other markets” (Böhringer & Löschel, 2006, p. 136). This fact emphasizes the failure of bottom-up models to represent the linkage between energy demand and the economic forces ultimately driving the demand in an adequate manner, what points to probable benefits of the hybrid structure adoption<sup>7</sup>.

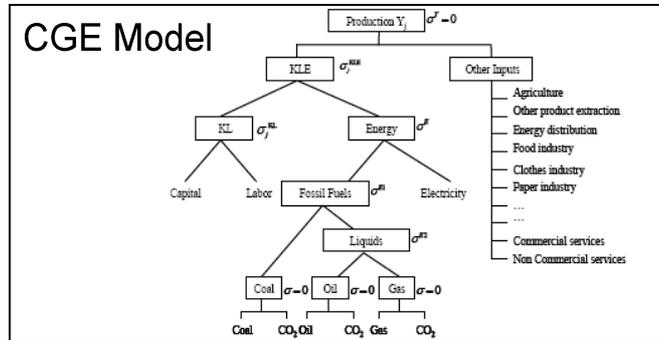
The first alternative to provide a more detailed representation of the energy sector production cannot be considered a hybrid model in a ‘*stricto sensu*’. It consists in formulating a top-down CGE model with detailed energy demand decisions represented

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<sup>7</sup> As an illustration of some applications that benefits from the hybrid approach we have: the treatment of (energy) “tax interaction and tax recycling effects (e.g. Goulder 1995), terms-of-trade spillovers on international markets (e.g. Böhringer and Rutherford 2002), or induced technological change (e.g. Otto et al. 2006)” (Böhringer and Löschel, 2006, page 137).

directly by economic production functions with n-nested levels and specific technology substitution elasticities (see Figure 2). The EPPA-MIT model (Paltsev, et al., 2005) and the work of González-Ruiz de Eguino (González-Ruiz de Eguino, 2007) are examples of the utilization of this approach.

Figure 2: Energy sector detailed in a pure CGE formulation.

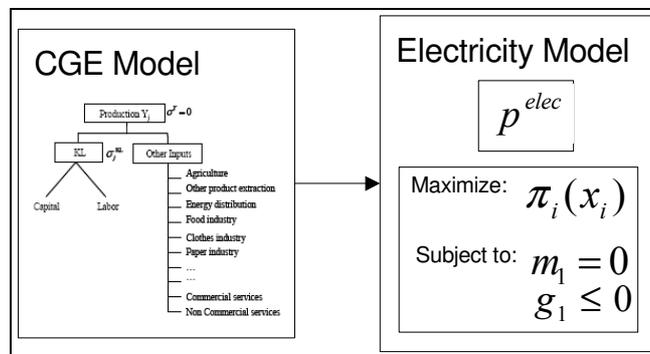


Source: Own elaboration.

The construction of a reduced-form sectoral model according to the CGE tradition adds few complications to the original data requirements. Besides, it is possible to utilize an independent bottom-up model to calibrate its own elasticities. The model from Drouet et al. (2008) follows this approach, using a nested CES reduced-form model, involving capital, labor, energy and materials, with elasticities estimated by detailed partial equilibrium models for electricity, transportation and industrial sectors. Again, the same approach is taken by Pizer et al. (2006) when addressing an economic analysis of climate change policies.

Another hybrid alternative is assembling a soft-linking approach (see Figure 3). A soft-linking approach employs sequential models to obtain a solution, i.e., soft-linking involves generating outputs from one model to serve as inputs to another model without physically connecting the two. As Mitra-Kahn pointed out, the “idea of having a ‘chain of models’ where one a set of exogenous variables would be endogenous further down the chain, was formulated in Robinson (1976) and described in Adelman and Robinson (1978).”...”Adelman and Robinson were the first to link CGE models in a chain, and this idea has become very influential since.” (Mitra-Kahn, 2008, pp. 27-28).

Figure 3: Softlink, sequential, hybrid formulation.



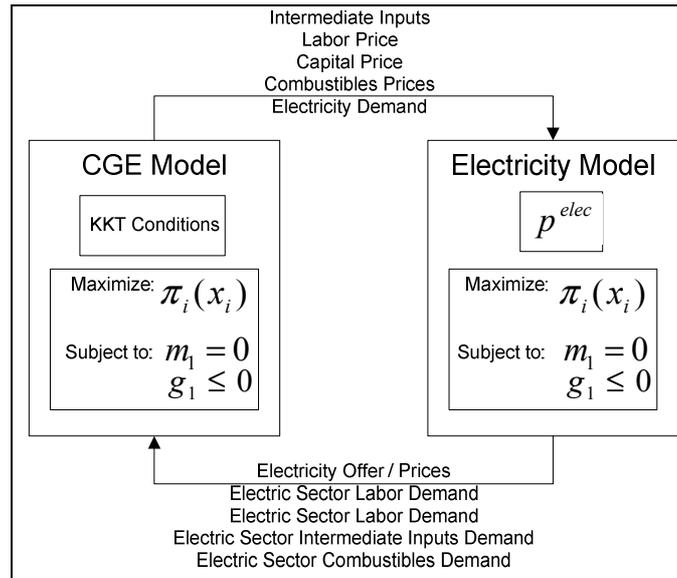
Source: Own elaboration.

A sequential soft-link approach allows without many additional data requirements to explore in more detail the parameters determined in the first model applied. Nonetheless, this formulation does not consider entirely the cross influences of demand

and income effects contained in CGE formulation. The works of Wene (1996) and Labandeira, Labeaga, & Rodríguez (2006) could be considered as examples of models with a soft-link formulation. Wene specifically makes an analysis about a soft-link approach of a bottom-up engineering model called MESSAGE and a top-down macroeconomic model called ETA-MACRO.

Nevertheless, in order to take advantage of the cross and indirect effects between the economic models it is necessary to implement a feedback instrument connecting both, as in an improved soft-link approach (see Figure 4).

Figure 4: Soft-link, with feedback, hybrid formulation.



Source: Own elaboration.

The idea consists in iteratively linking the two models until a convergence is reached. Turton (2008) makes use of this approach in ECLIPSE and MESSAGE-MACRO models linkage, while Böhringer & Rutherford (2006) present a similar iterative decomposition and Linares, Rodríguez & Labandeira (2008) evaluate the effects on Spanish economy of carbon policies based on European trading scheme applying a general equilibrium linked with a detailed bottom-up electricity model.

In Turton's (2008) model, the basic economic model considers separately the output of the energy system (production of energy and transport) and the output of the rest of economy. That way, information about energy and transport produced in the bottom-up part of the model (ERIS) are necessary to obtain the results for the macroeconomic model, however, in contrast to a simple soft-link approach, the linkage is made in an interactive way. A simulated cost function determined by bottom-up dependent's parameters is applied to the macroeconomic model. The solution to these parameters is obtained by iterating energy demands into ERIS, which determines the energy shadow prices that are then fed into the macroeconomic model, which determines new demands. This process is repeated until convergence criteria are satisfied.

In the case of a bottom-up model formulated as Turton's, the full integration between the models would require the construction of the energy cost functions implied in the bottom-up model for each possible point along the supply curve, which would require an impractical computation for the complete integration of bottom-up and top-down

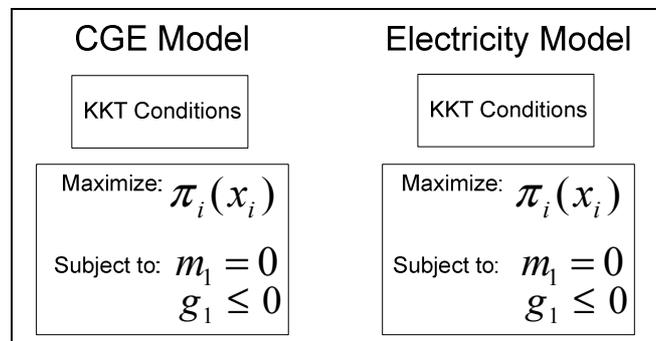
models. As trade-off, such models have particular obstacles in the achievement and assurance of a convergence level.

The best of both worlds – top-down and bottom-up integration – would only be obtainable through a hard-linking formulation, i.e., physically connecting two or more models. However, there are several differences between the formalized language describing bottom-up (partial equilibrium models) and top-down (CGE models) that raises some difficulties to make their linkage.

In order to deal with this problem, Sue Wing (2008) presents a detailed procedure formulation for disaggregating the top-down macroeconomic representation of the electricity sector in the sub-activities that integrate the sector –generation, transmission and distribution – in a manner consistent with the characteristics of bottom-up engineering technologies.

A real integration can only be obtained by the formulation of the bottom-up structure in a similar input procedure when compared with the CGE modeling, especially in terms of the production factors utilized. Implementing this step, it is possible to substitute the elasticity economic-technological description by a more realistic and rich bottom-up formulation. Therefore, rather than describing the production technologies in form of many levels of CES production functions, the production possibilities could be described as in detailed engineer bottom-up models and inserted in CGE formulation through the utilization of Leontief technologies, utilized depending of their profitability, or even directly inserted if the output data of each model is compatible.

Figure 5: Hard-link, mixed complementarity problem, hybrid formulation.



Source: Own elaboration.

The linkage adopted by the work of Böhringer & Rutherford (2008) is the one that more resembles a hard-link approach, where the solutions to the models are obtained simultaneously through a mixed complementary problem (MCP) (see Figure 5). The Karush-Kuhn-Tucker conditions of the CGE equilibrium model and of the bottom-up engineering optimization are incorporated in a unique non linear equilibrium problem.

Even though, the linkage with the top-down model would be done directly only if the cost functions in the bottom-up representation were compatible with the primary economic factors utilized in the top-down structure. Consequently, it is required that the inputs of each bottom-up technology be described in compatible data with CGE factors and costs, and vice versa. In the electricity sector for example, the SAM should present the energy sector (transmission, distribution and generation) and the demand for fuel consumption disaggregated; at the same time, the bottom-up model should refer to labor participation in operation and maintenance costs, to capital costs in equivalent overnight

investment costs and to the prices of specific fuel sectors described in the SAM as the bottom-up fuel costs.

The information required for this method is not easily translated from the SAM because of the aggregated format of its data, which make it difficult the determination of the labor and capital share of each production technology type. Even when it is possible to do this, the calibration of future high-cost “backstop” technologies that are currently unprofitable would be impossible without additional assumptions. As well, the MCP formulation can need additional computational requirements and represent a consequent limitation to the bottom-up model size.

### **V. c. Dynamic CGE**

The descriptions made until now were centered in static CGE formulations. These formulations are popular in CGE modeling because they emphasize the impact of allocating resources across sectors of an economy, identifying winners and losers under a policy change (Kehoe & Kehoe, 1994). However, they fail to capture the dynamic aspects of a policy change bringing therefore the need for alternative dynamic CGE formulations.

Classifying CGE time-dependable models as dynamic models is a controversial issue. As Mitra-Kahn describes, “‘dynamics’ in CGE models is a number of static equilibrium solved one after the other, and not a dynamic process nor result of any kind, meaning they are no more dynamic than the standard Solow growth model” (Mitra-Kahn, 2008, p. 72), meaning that CGE models are actually ‘pseudo-dynamic models’, mostly defined as a recursive succession of static models.

Nevertheless, setting aside the nomenclature matter, the central issue of a ‘dynamic’ CGE model is to analyze the temporal evolution of the model variables according to a specific accumulation and technological progress. Phenomena like capital flows, demographics, economic growth rates, and even technological progression over time can be analyzed in such framework.

The first concern that a ‘dynamic’ modeler should address is the determination of the capital (investment) accumulation process, i.e., the modeler should determine the actual drivers of economic growth on the model formulated. As in the previous case of savings-investment balance closure (subsection V. a), a series of theories can be translated from economic growth theory to the CGE formulation (see Table 4). Nonetheless, as in previous issues, the standard assumption assumed in CGE modeling is the utilization of traditional neoclassical theory to describe the growth process.

This table draws heavily on the contents of the work of D’Agata & Freni (2003), and as an overview incurs in possible arguable and incomplete descriptions. It is strongly recommended to examine D’Agata & Freni (2003) original work in order to obtain more details on the authors and theories cited above.

Table 4: Taxonomy of economic growth theory.

Growth Theory	Steady State	Growth Driver Variable	Mechanism driver	Major authors	Observations
Classical	Tendency towards the stationary state.	Savings	The rate of growth of the economy is determined by the interplay between savings and population growth rate, the former being completely employed in investment and the latter being endogenously given as an increasing function of the real wage rate.	Ricardo Smith Marx	Classical economists assume that different savings/consumption propensity for different agents (workers, renters', capitalists) cause different savings amounts depending of the economic distribution. Ricardo put emphasis on the role of natural resources as a factor limiting growth.
	An economy does not find full employment and stable growth rates naturally. Steady state occurs as an accidental, very specific alternative.	Investment	In the Harrod-Domar model, investment is determined independently from savings by long-term profit expectations and, to a lesser extent, by the interest rate. More investment leads to capital accumulation, which generates economic growth. In the Kaldor and Pasinetti perspective, economic growth is determined by the attitude towards investment of the society and, in particular, of entrepreneurs.	Harrod-Domar Kaldor Pasinetti	The proposition that investment determines savings intrinsically connects the Keynesian theory of growth with the theory of business cycle. Kaldor (and after Pasinetti) maintain the fundamental Schumpeterian intuition, that a satisfactory growth theory cannot be constructed without a business cycle theory, and follows the Keynesian approach in conceiving the expansion of the economy as driven by psychological and social factors like 'human attitude to risk-taking and money-making'.
Neoclassical	Factors full employment. Convergence to a natural growth path.	Savings	Growth is affected only in the short-run as the economy converges to a new steady state output level. In the Solow model the growth level is determined by the technical progress and the population growth.	Solow Ramsey	The neoclassical exogenous growth theory is a theory of evolution of the potential output, rather than a theory of actual rate of growth as the Keynesian approach. The Ramsey model is equivalent to the Solow model, though production and savings are decided by a planner choosing over an infinite horizon.
	Growth path stable as product of the endogenously treated resources.	Savings	All variables which are crucial for growth – in particular savings, investment, and technical knowledge – are endogenously treated, and determine the growth path. The path is usual obtained as an outcome of a rigorous model through rational decision.	A-K Lucas (Human Capital) Romer (Technical progress)	The savings dynamics usually follows Ramsey's (exogenous growth) scheme, however the endogenous resources, like human capital or technical progress, determine the growth path.

Source: Own elaboration.

The neoclassical theories of growth include most of the classical school paradigms, improving specific selected fields and refuting especially the classical theories of value and labor (especially the Marx theory of value). As previously mentioned on the static case of Walrasian equilibrium, their assumptions include the tendency for full utilization of all production factors through combination of factors substitutability and price flexibility concepts.

In a broader sense, these neoclassical growth models fall into two major categories: the exogenous and the endogenous growth models. Pure exogenous models are based on the assumption that economic growth is obtained through parameters that are independent of the actual modeled variables, i.e., economic decisions embedded in the model do not change the potential outcome of the economy, which is in fact determined by exogenous parameters such as population growth, natural resources availability or technological limitations.

The principle underlined is that under a capital accumulation process with diminishing marginal returns, the rate of growth is exogenous in relation to the rate of savings and investment, and depends of exogenous variables as labor force growth or technological progress.

The result of such models is the existence of a steady state path of economic growth defined by the values of the exogenous growth drivers. Moreover, under the absence of market imperfections and, principally, under the presence of a full employment hypothesis there is a nonstop force that acts on forcing the convergence of the economic output to the steady state level.

The majority of dynamic CGE models follows this approach and is based in economic growth models à la Solow (1956), and, above all, à la Ramsey (Ramsey (1928), Cass (1965), Koopmans (1965)).

In contrast, the neoclassical endogenous theories of growth assume the existence of an accumulative production factor which does not present diminishing marginal returns. Consequently, an investment increase in this factor could boost the equilibrium rate-of-growth of the economy changing the steady state path to a new future level.

The A-K models (one sector models) presume that the physical capital can be accumulated under constant marginal returns, and by this, the labor productivity increases in the same proportion of capital depreciation. The constant rate 'labor-production/capital-depreciation' cancels the tendency to diminishing marginal returns and determines the possibility of higher rates of investment being associated with higher rates of growth for both the product and the product per capita.

In Solow models (1956), the physical capital still possesses diminishing marginal returns (as in exogenous growth models), however, an economic sector is responsible for the production and accumulation of a specific production factor under constant marginal returns, the 'knowledge', responsible for raising the stock of 'human capital'. As a result, there is a trade-off where: a present effort in the production reduction of other sectors in favor of an increase in the production of knowledge raises the long-term growth rate of production and the long-term growth rate of product per worker. The same process can be described when dealing with endogenous technological progress as in the Romer (1986) (1990) perspective.

While the assumptions contained in neoclassical economic growth theories follow the traditional CGE formulations and allow for the incorporation of capital temporal evolution in such models without severe complications, other alternatives still remain.

Accordingly, economic theory includes several alternative growth theories that should not be set aside without careful evaluation. Most of them focus specially in the refusal of the neoclassical supposition of an automatic tendency to full employment (of production capacity utilization and of factors as labor), and in the adoption of an asymmetric determination of the income distribution between supply and demand (i.e., social distribution matters). In such heterodox theories, as there is no complete utilization of the labor force, the economy potential product is dependent of the capital stock and its efficiency, and so, the production capacity growth depends on the time evolution of investment. The determination of investment, in turn, is largely discussed and particular to each theory. Classical and some developmentalist authors, for example, assume that the potential savings would be the necessary and sufficient condition for investment in the long-term (following the Say's Law tradition), while other authors follow Marx, Keynes and Kalecki in the notion that investment decisions are independent of savings decisions and determine the aggregate amount of savings held (effective demand principle) (Serrano, 2008). Either way, the dynamic evolution of the capital stock, and economic production levels, through time should be clearly identified by the CGE modeler.

- **Technological Change**

In the technological sense, as Schumpeter (1961) expressed, economic growth can be lead by a process of innovations and innovative activities. The economic dynamics would then include in its determination a process accompanied with disturbances from the actual economic equilibrium. These disturbances would be caused by the action of an entrepreneur in the development of an innovation that ensures economic advantages. The entrepreneur's actions trigger an innovation process connecting an uncertain emergence of an innovation as the instigator of economic development, and the driver of economic cycles.

Per se, CGE models, even if accepted as a reasonable economic description, are only capable of reflecting a stationary state assuming *ceteris paribus* technology levels. Therefore, such models are incapable to represent the already expressed technological evolution process<sup>8</sup>.

However, the majority of models developed, especially the ones in the engineering tradition, address assumptions referring to predictable behavior of the future technological situations, usually related with the creation of discretionary future scenarios. CGE models do not differ in this. They include not only actual production process descriptions, as stressed in subsection V. b, but also assumptions about their evolution in time as will be portrayed in the following paragraphs.

The first and simpler way to introduce technical change in CGE models is through an exogenously defined technological change factor. Future production costs, technological

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<sup>8</sup> See section VI. b, especially in regard to the concept of strong uncertainty to understand the impossibility to a model describes the future consistently, and consequently to describe the entire technological progress consistently.

availabilities, productivity increases, scarce resources quantities, etc, necessary to the formulation of specific dynamic CGE models are usually treated in the same way that GDP, population growth, unemployment, discount rate and exchange rates levels are treated in more general CGE models; i.e., their future levels are exogenously determined by a decision based in the discretionary analysis of the modeler.

A revision of the exogenous levels assigned to these variables allows for the evaluation of a CGE comparative static analysis of specific exogenous shocks or spillover effects in the economy studied. In energy CGE models, it is also typical to assume an exogenous definition of variables like the energy use per unit through time, the so-called autonomous energy efficiency improvement index (or AEEI) as cited by Grubb et al. (1993) and utilized in MIT-EPPA model in Jacoby et al. (2006).

However, an alternative to the predetermined future levels is to model specific technical conditions and let the model 'pick' the optimal choices for each situation. Through this method, the internal specification of the model is responsible to select the most propitious between alternative technologies, facing different relationships between the production factors and the allocation of resources current presented in the models.

This alternative, represented by endogenous technical change, cover diverse possibilities: technological choices caused by the shortage prices and quantities of resources utilized in specific technologies are very common in bottom-up models, and correspond to situations like bringing into action technologies initially not competitive, but improved by the effect of changes in fuel costs (this effect depends directly on how it is shaped the future price of fuel scarcity) (Linares, *et al.*, 2008); application of different learning curves of efficiency and technology adoption that cause changes in the elasticity of substitution for different technologies in different periods; the already cited Knowledge accumulation (via R&D), which could provide a temporary monopoly power to the firm owner of the innovation until dissipating effects come into action (like spillover effects or competitors entry in the market), or more directly, the effects caused in production efficiency and costs of progressive knowledge obtained in 'new' technologies like the representation of advanced biofuels in the WITCH model (Bosetti, Massetti, & Tavoni, 2007) (Bosetti, Carraro, Galeotti, Massetti, & Tavoni, 2006).

Moreover, endogenous technological changes can be implemented in hybrid modeling either in the bottom-up as in the top-down branches. Taking the learning-by-doing example, top-down models could incorporate learning curves of adoption on the most efficient response mechanisms of consumer demand, as in the active demand response issue (DOE, 2006) (FERC, 2006) (Rocky Mountain Institute, 2006), or curves of increase in energy efficiency subordinate to the model technological choices. At the same time, bottom-up models could address issues like technological power generation and expansion choices in electricity models, with respective learning curves based in utilization of the technology. This approach of simultaneous mutable technology in both bottom-up and top-down sections allow a CGE analysis about the temporal gains of policies that encourage more rapid adoption of more efficient processes, whether in consumer or productive sectors' behavior. However, it implies also data requirements not easily obtainable.

## **VI. Limitations and critiques**

Once presented the components and the theoretical alternatives embedded in a CGE formulation, it is finally possible to assess the limitations of this policy evaluation instrument. In order to achieve this limitation assessment, this section is divided in two subsections: the first refers to the mathematical limitations embedded in the resolution of CGE models and the second refers to the economic boundaries, properties that are unfeasible in such models.

### **VI. a. Mathematical limitations**

CGE models are deterministic non-linear systems of equations. Nonlinear equations are usually difficult to solve and can involve problems such as: indetermination and nonexistence, multiple equilibrium's, instability, or even multistability properties.

The indeterminate problem occurs when the system of equations cannot be solved, given that the unknown variables are more numerous than the independent equilibrium equations. However, enough independent equilibrium equations do not ensure that equilibrium exist for all times. These facts raise the necessity to evaluate the conditions for equilibrium existence in CGE modeling. Additionally, non-linear systems can have multiple isolated equilibrium points, raising the necessity to a uniqueness analysis of the solution, and the possibility of an instable, or multistable, system of equations.

In the existence subject, CGE models are calibrated to reflect an initial equilibrium point; however, this equilibrium point is insufficient to guarantee the computation of a new solution after changing a specific parameter, bringing the necessity to evaluate the model's equilibrium existence.

The conditions of equilibrium existence for a non-government closed-economy Walrasian competitive equilibrium, as the one presented in subsection II. a, involves assumptions such as: the convexity of the consumer preferences and of the feasible production sets, monotonicity, divisibility, perfect-competition, complete information, etc.

Many of these assumptions can bring tough consequences in the model formulation. For example the convexity of feasible production sets excludes the possibility of economies of scale. Although the majority of CGE literature disregards the existence demonstration, as pointed out by Kehoe et al. (2005), Ginsburgh & Keyzer (1997) made a compendium on applied general equilibrium that comprise the discussion of existence of equilibrium under different postulations.

As demonstrated by Ginsburgh & Keyzer (1997), it is still possible to obtain a proof of existence through a fixed point correspondence when assuming an open economy CGE model. The same occurs with the inclusion of government (taxes, tariffs and quotas), since the government representation can affect only budgets and price formation equations, setting unchanged equations referring to balances for goods, factors and imports. Price rigidities, finite and infinite horizon dynamics and externalities require more specific assumptions related to bounded variables, decreasing returns to scale and others. Finally, imperfect competition opens several possibilities for the model formulation and consequently to the analysis of equilibrium existence. However, a proof

of existence is clearly obtainable under a markup pricing rule representation since this rule can be treated as a simple commodity tax.

Once solved the existence problem, ensuring uniqueness of equilibrium is a much more arduous problem, even under the simplified general equilibrium model showed in subsection II. a. Moreover, according to the Sonnenschein-Mantel-Debreu theorem, only a certain number of microeconomic assumptions (continuity, homogeneity of degree zero, Walras's law, and boundary conditions) are maintained in aggregate excess-demand functions, this means that microeconomic rationality assumptions have no equivalent macroeconomic implications.

Under these strictly maintained properties, there is no guarantee that the Weak Axiom of Revealed Preference<sup>9</sup> (WARP) would be preserved in aggregate, even if individual demand functions satisfy the WARP. Consequently, the possibility of multiple equilibrium existence is inherent to macroeconomic models formulated from microeconomic assumptions.

In economic terms, the non-equivalence between microeconomic suppositions and macroeconomic implications is due to the presence of income effects. As explained before, every price change implies two effects, a substitution effect (feature presented in both partial and general equilibrium models) and an income effect (explicit feature of general equilibrium models). The offset or reinforcement of both effects makes it possible for more than one set of prices to constitute an equilibrium.

The complication inborn from the possibility of multiple equilibriums added to their potential instability, points out to the necessity to analyze if the equilibrium is at least locally unique. Due to this reason, it is crucial to formulate a sensibility analysis of the obtained CGE equilibrium point in order to be able to apply the model in comparative statics analysis, as long as the shocks to the system are not large enough to involve substantial changes.

An additional concern should always be kept in the mind of the modeler. Even that, under preferences locally nonsatiated, a competitive equilibrium is always Pareto efficient (first fundamental theorem of welfare economics), nothing ensures that a specific equilibrium under a multiplicity of possibilities is the optimal solution to a secondary variable not directly evaluated by the model. That means that even under a Pareto efficient result there is no indication if the chosen situation is one that overestimates or underestimates, for example, the environmental or the economic distributional outcome. This fact brings complications to the determination of the social gain of a specific policy, the calculus of welfare, as for example in cases where the production and emissions levels are evaluated simultaneously because an increase in poverty (less electricity intensive) could also cause a reduction of emissions. Consequently, there is no such thing as a 'complete general equilibrium model' and the CGE model proposed should always, as much as possible, express explicitly and in detail the policy evaluated relating secondary goals by its social benefits or, more commonly, maintaining then *ceteris paribus*.

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<sup>9</sup> The Weak Axiom of Revealed Preference states that if A is ever chosen when B is available, then there can be no optimal set containing both alternatives for which B is chosen and A is not.

## VI. b. Economic boundaries

CGE models are capable of representing different economic characteristics either through: closure options, dynamic alternatives, description of technology, or microeconomic choices. This provides the ability to describe not only pure neoclassical models, but also variations that incorporate alternative economic growth assumptions, market imperfections, neo-Keynesian postulations<sup>10</sup>, and even a partial representation of the structuralism school assumptions<sup>11</sup>. However, the embracing of a deterministic equilibrium model implies a series of unattainable economic characteristics, undescrivable in the model formulation.

A straight criticism can be addressed by the limitation of CGE models in representing fundamental heterodox assumptions. It is impossible to represent a truly Keynesian economy in such models, either by the impossibility to represent the uncertainty or by the limited representation of the demand behavior.

Tautologically, deterministic models do not model random behavior, and as such are incapable to directly model not even measurable risk/uncertainty<sup>12</sup>. A common way to go around this problem is applying the CGE deterministic approach under a group of scenario evaluations or Monte Carlo simulations (Webster, et al., 2002). At the same time, an alternative to handle endogenously this kind of ‘weak’ uncertainty is the adoption of a different modeling approach named dynamic stochastic general equilibrium modeling (DSGE). In DSGE models, some important economic parameters, as GDP, consumption, investment, prices, wages, employment, interest rates, between others, are estimated using Bayesian statistical techniques in order to approximate their levels to the observed behavior, but still making use of microfoundations in the determination of agents’ behavior.

In a more strict sense, there is no model, neither CGE nor any other, capable to model the immeasurable uncertainty that could not be reduced through an objective probability

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<sup>10</sup> Neo-Keynesian postulations include microeconomic models designed with imperfect competition, asymmetric information and limited rationality (Joseph Stiglitz), besides price ‘stickiness’ (giving emphasis to quantity adjustments early than price flexibility).

<sup>11</sup> The structuralism economic school (Taylor (1990) and (2004)) is based in the description of the economic process through the focus in the economy structural components and their actual situation, rather than how the economy is conceptualized in theoretical terms (neoclassical approach). This means that their economists focus their analysis on system-wide analysis and in the social structure of the economy prior to its individual’s parts. The emphasis is consequently given to the macroeconomic analysis, but such models are still capable of utilizing microfoundations, usually related with the Kaleckian tradition (1971) (microfoundations based on the degree of monopoly, in a macroeconomic dynamic that incorporate tendency and cyclical behavior, with a succession of temporary equilibriums, founded around the principle of effective demand).

<sup>12</sup> Under this work perspective, there are two types of uncertainty: a ‘weak’ uncertainty and a ‘strong’ uncertainty. The ‘weak’ uncertainty represents the probabilistic risk of a specific economic decision, this means, a measurable level of risk/uncertainty obtained either through statistically estimable risk by frequency probabilities or through interpreted risk level given by an axiomatic probabilistic determination. The ‘strong’ uncertainty corresponds to the uncertainty in Knight-Keynes hypothesis that means merely that we simply do not know the future, i.e. it corresponds to the portion of the future that is unpredictable and impossible to modeling.

distribution; the strong uncertainty in the Knight-Keynes sense. This drawback embedded in economic models sets a direct limitation to the admission of the model results and opens the field to a common, and even necessary, discretionary ex-post evaluation of the validity of the outcomes obtained by the model.

Without uncertainty and assuming a maximizing behavior, production and demand decisions reach a perfect match. There is no possibility of economic transactions occurring under less than the efficient full capacity utilization, and there is no room to errors in the demand expectations of the production process.

Under this scenario, the production decisions are fully compatible and completely absorbed by the demanders'; besides, the income cycle presents no losses. This means that Say's Law is satisfied; the production creates its own demand. Consequently, the production sector characteristics and technological limitations have a much more important role in the determination of the economic variables, while income plays a secondary role, a merely allocative effect, being incapable of affecting the production level of the economy.

The Keynesian and Kaleckian theories present a much more important role for the demand on the determination of the production level. According to Keynes, the productive agents contemplate a production decision (short-term production decision) and a decision of alteration of the company assets (long-term investment decision). In order to determine the decision of how much to produce and invest, expectations related with future sales are taken into account. As a result, the ex-ante decision of production is based in an expectation of the effective future level of demand, and the ex-post income from sales is determined by the demand that will be actually observed in the market in the future. Consequently, the determination of production levels, and employment, is made ex-ante, while the income, and the residual profits, are determined ex-post. This is the same as to say that the expenditure decisions (consumption and investment) determine the income, and therefore, the causality encountered in Say's Law is inverted. In such uncertain production systems, Say's Law is substituted by the principle of effective demand, which attests that in a mercantile economy the agents are only capable to decide how to spend (they are incapable of determining their own income). Thus, it is the demand who determines the supply, and accordingly, it is investment who determines savings.

In an equilibrium model, such as a CGE model, the simultaneity implied in the market clearing makes impossible the description of uncertainty effects. Consequently, the effects obtained from the inversion of the causality between supply and demand are suppressed. There is always a perfect match between production and demand decisions excluding the uncertain confirmation of the production expectations and their contribution to economic cycles<sup>13</sup>, which most probably oversimplifies the economic process in such models.

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<sup>13</sup> It is still possible to represent specific types of economic cycles in such models, as the ones described in real business cycle theory, mostly based in variations of product stocks. These real business cycles do not impose a failure of market to clear (as opposite to the Keynesian cases of excess or insufficient demand effective under uncertainty environment), but actually reflect the most effective operation of the economy through time.

The assumption that the economy remains in equilibrium at all times is a vital point in the criticism of CGE models, and also in the criticism of neoclassical theory. In this perspective, the equilibrium is proposed as a synonymous of solution. However, in Kaldor's (1972) view, "the markets of the real world are not in continuous equilibrium in this sense; there are, or can be, persistent differences between production and consumption which are reflected in increments or decrements in stocks" (Kaldor, 1972, pp. 1247-48), and as Vercelli (1991) pointed out, the solution of a system of equations is simply one set where this system is logically possible, and as a result, the non-equilibrium situation should also be analyzed in a consistent economic theory. Only a CGE approach that incorporates a non-balancing persistent mechanism that allows absence of market clearing could endogenously account for issues like that.

An additional relevant point is that every CGE model is a relative prices model (a real economy model), and as such, assumes a neutrality of money in the economy. As such, the assumption that money and liquidity matter, assumed again by Keynes (1936) – theory of liquidity preference –, besides other economic schools as the Austrian school – where the non-instantaneous adjustments in monetary stocks cause effects in the economy because of their influence in the conditions of trade and production – are impossible to be represented in CGE models, directly invalidating their application in more complex issues related with money like inflation patterns and estimation of future price levels.

## **VII. Conclusions**

The state of the art on CGE modeling has been presented. The subject is motivated by the difficulty to evaluate alternative economic policies, their consequences and their repercussions, on a complex environment such as the real world.

CGE models can be an auxiliary assessment tool for ex-ante simulation of the adjustment effects induced by exogenous policy interferences. As described in section III, the alternative partial model representation is not capable of representing entirely the substitution and income effects that arise from market iterations. The general equilibrium model can deal with these effects, although unfortunately, this advantage does not come without trade-offs.

Economic assumptions traditionally found in CGE models like homogeneous capital, flexible prices, steady-state growth path and perfect-competition markets can make their results deviate from reality, and even produce arguable cause-effect theoretical chains. Meanwhile, mathematical problems as the existence of multiple equilibrium's, instability and even nonexistence of equilibrium arise under more realistic economic assumptions.

Decisions of fiscal, commercial and environmental policies, structural adjustments and energy sector strategies can be improved through the use of auxiliary economic models such as a CGE. However, it is essential to notice the lack of capability of such models to predict future economic values accurately. CGE models are not designed, in principle, to predict the behavior of variables, and as such will never provide an empirically verifiable result. Their major objective should be to describe in a methodical formulation the cause-effect economic relations and their repercussions on economic variables. That is to be a model capable through an abstraction of reality to capture the

essential features of an economic circumstance, while also providing a tractable theoretical model for policy evaluation.

Therefore, a robust Computable General Equilibrium formulation should: explicit the set of microeconomics assumptions accordingly to its specific application – for example, non transient market structures in the studied time horizon should not be overlooked, as the case of imperfect competition caused by a market structure of market power (persistent in a short-medium term analysis) in sectors like the energy/electricity sector –; explicit and justify the macroeconomic closures and, for the appropriate case, the ‘dynamical’ accumulation process chosen for the economic simulation; evaluate the accuracy and reliability of the technological descriptions embedded within the model; evaluate the existence and stability of a solution, especially in multiple equilibrium’s situations, principally through the use of sensibility analysis; and finally always keep in mind the limitations embedded in the endeavor of representing complex human and physical relations through the utilization of simplified mathematical models.

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### **3. RESEARCH WORK II**

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#### **INTRODUCTION**

This section contains the research work II entitled:

**PRODUCTION AND EMISSIONS IMPACT OF HOUSEHOLD ELECTRICITY  
DEMAND RESPONSE: A CGE ASSESSMENT FOR SPAIN**



# PRODUCTION AND EMISSIONS IMPACT OF HOUSEHOLD ELECTRICITY DEMAND RESPONSE: A CGE ASSESSMENT FOR SPAIN

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## **Abstract:**

Alterations of demand levels in electricity markets can bring substantial shifts in their production structure, costs, and level of emissions. Nevertheless, the electricity sector is not the only one affected as these changes can create significant repercussions in other sectors and, consequently, in the whole economy. In this paper, the indirect effects of reduction in household demand for electricity were evaluated for the Spanish market. A multisectoral static computable general equilibrium was employed to achieve this objective. The results clearly point out the importance of assessing other sectors behavior when valuing the consequences of fomenting demand response policies, especially when dealing with emission objectives.

**Keywords:** Computable General Equilibrium (CGE), Emissions, Electricity Demand Response

**JEL classification:** C68, D58, Q4, Q51, L60

## **I. Introduction**

Electricity systems face important challenges in order to decrease emissions. One of these challenges has to do with peak demand, which involves considerable economical, environmental and technical inefficiencies, which in turn arise from the necessity of available infrastructure with a low utilization factor. The technologies applied to supply peak demand present higher variable costs and lower fixed costs. Moreover, nowadays these technologies are based in fossil fuels, which are usually highly polluting.

In this challenging context, Demand Response (DR) programs have gained importance in the last years as one of the options to smooth and adjust the consumption profile and as a consequence decrease the need for highly polluting technologies. DR programs intend to facilitate customers' reaction to the technical and economical needs of the electrical system. Receiving an effective price or quantity signal, customers would have an incentive to reduce their consumption in periods where the prices reflect situations of more inconvenient production for the system. In electricity markets, DR programs have two effects, downsizing the demand levels and/or cause its displacement through time. Secondly, DR programs could also have other advantages for the integration of renewable energy, distributed generation and electrical vehicles or in increasing customer's awareness of their consumption.

The evaluation of the impact of a DR program is then an important issue that walks side by side with the necessity to acquire an understanding about their consequences and about the correct signals that should be provided to consumers and productive sectors before actually engaging its implementation. Borenstein et al. (2002) and Boisvert y Neenan (2003) started with this evaluation by showing an analysis of DR consequences in a theoretical way; however, quantitative measures are also necessary to evaluate the impact of such policies. At this field, the most usual approach adopted in the literature makes use of partial equilibrium models. Berg et al. (1983), Caves et al. (1984), Parks & Weitzel (1984), Hill (1991), Borenstein (2005), Andersen et al. (2006), Holland & Mansur (2008), Brattle Group (2007) and Conchado & Linares (2009a) are all examples of works that evaluate the social or environmental cost-benefit of an increase in DR, or similar issues, through diverse models under the partial approach paradigm.

However, the partial equilibrium approach disregards the impact of the interactions of variables concerning the others sectors of the economy. For instance, it is often assumed that fuel prices and demand levels are exogenous variables, which means that in this kind of model the interaction between these variables and the electricity market is not taken into account for the clearance of the electricity market.

Therefore, partial equilibrium models consider only one market at time, and deal with sources of linkages across markets, like wealth effects, exclusively exogenously. Nevertheless, in order to evaluate policy interventions that affect large numbers of markets simultaneously these linkages cannot be neglected. One could consider DR programs as one of these cases because of the strong weight of the electricity sector in the determination of economic levels, its huge interrelation with other productive sectors, and its significant environmental influence. A general equilibrium approach is then necessary to address this issue correctly.

This study evaluates the impact of changes in the DR of Spanish household electricity consumption and its consequences not only directly related to the electricity sector, but also to the entire country economy. For this, it makes use of a general equilibrium formulation. The resulting model views the economy as a closed and interrelated system in which the equilibrium values of all variables of interest must be determined simultaneously.

Computable general equilibrium (CGE) models have been applied as a tool to assist economic decisions since the early 1970s. Evolving from Leontief's 1930s multi sector input-output models, CGE models have been presented as an alternative tool for economic evaluation since the seminal works of Johansen's (1960) and Taylor & Black (1974). Several compendiums indicate the rapid development of empirical CGE applications in a variety of policy, energy and environmental issues. Lewis & Robinson (1986) and Decaluwe & Martens (1987) grouped numerous models from diverse countries with different modeling objectives that use CGE modeling. Gómez-Plana (2002) offered a revision of CGE models applied to Spain; and Ginsburgh & Keyzer (1997) offered a CGE survey oriented by diverse theoretical assumptions.

Ever since, empirical CGE models have been applied in topics covering fiscal, commercial and environmental policies like: international trade (Taylor & Von Arnim, 2007), public sector and goods (Bernow, *et al.*, 2002), agriculture planning (Wittwer, *et al.*, 2005), income distribution (Bandara, 1991), development policy (Dervis, de Melo,

& Robinson, 1982), growth and structural adjustment (Robinson, *et al.*, 1993), energy efficiency and sustainability (Hanley, *et al.*, 2009), environmental issues and global warming (Böhringer, Löschel, & Rutherford, 2006).

More recently, an increase interest in developing models capable of evaluating energy and environmental policies provided incentives to the formulation of a series of multitask large models, like the GEM-E3 model – European commission (Capros, *et al.*, 1995) –, the GREEN model – OECD (Organization for economic co-operation and development) (Burniaux, *et al.*, 1992) –, and the EPPA model – MIT Emissions Predictions and Policy Analysis (Paltsev, *et al.*, 2005).

However, despite its obvious importance, there is an absence of CGE empirical works to assess the economic impact of an increase in demand response. This paper intends to fulfill the first part of this gap by evaluating the demand reduction effects of DR programs, while additional research under work should be able to address the assessment of DR load displacement. In order to achieve this, the paper is structured as follows: section II presents a partial equilibrium analysis of DR that further in the work will be utilized as a comparative factor to the general equilibrium model. Sections III and IV describe the general equilibrium model and present and analyze the results obtained in the study. Finally, section V provides the conclusions drawn from the study and points to possible future extensions.

## **II. Partial Equilibrium Model**

Increasing the electricity demand responsiveness of the consumers causes a change in their consumption profile. It is reasonable to assume that an increase in the consumer awareness of the more expensive hours to buy electricity might cause a shift in their consumption habits in order to utilize less intensive equipments in these hours and avoid excessive payments. As the electricity prices are higher when the demand is higher, peak-hour demand should suffer the biggest effects from the DR programs. Then, logically, the direct result of DR programs is to cause a flattening in the consumption profile by transferring peak demand to less expensive hours.

Considering a partial equilibrium analysis, the relocation caused by the change between the peak and off-peak consumption results in a technological change in the structure of the electricity production. The flat profile would avoid the necessity of technologies with low utilization factors. As the trade-off for the flexibility offered by peak units, it is usually the presence of a higher operational cost and pollutants in these units; consequently, the increase in DR would cause a drop in prices and would promote the relative utilization of more “clean” electricity technologies.

The estimation of the manageable Spanish residential demand potentially influenced by DR programs is taken from the works of Conchado & Linares (2009a) (2009b) and numerically corresponds to a reduction of 6.61% of the total household electricity consumption.

- **Spanish electricity sector model**

The partial equilibrium model utilized to simulate the Spanish electricity sector under the DR scenario is the GEPAC model developed by Linares *et al* (2008). The model

incorporates a detailed representation of the Spanish electricity sector encompassing the oligopolistic structure of the electricity market, carbon emission markets and tradable green certificates.

The model provides the optimal operation and investment decisions of production to the Spanish electricity market. Its results assuming the new residential demand promoted by DR programs are utilized to provide comparison numbers between the different general and partial modeling approaches. The following table (Table 5) describes the most important figures obtained in the electricity sector DR GEPAC model.

Table 5. GEPAC results to an increase in DR. Units: MWh for quantities and €/MWh for prices.

		Initial value	DR scenario	
			New Level	Percentage
General Results	Household Demand	60971,2476	56943,2995	-6,61%
	Total Annual Demand	253384,854	249356,906	-1,59%
	Electricity marginal price	54,67	52,97	-3,11%
Technology (Generation Mix)	Nuclear	63037	63037	0,00%
	Carbon	73895	71968	-2,61%
	Fuel oil	1522	790	-48,09%
	Gas	160	23	-85,63%
	Combined cycle	38591	37378	-3,14%
	Biomass	6416	6416	0,00%
	Cogeneration	19254	19254	0,00%
	Mini-hydraulic	4680	4680	0,00%
	Wind	15996	15996	0,00%
	Solar	18	18	0,00%
	Manageable Hydraulic	17906	17906	0,00%
Flowing Hydraulic	11870	11870	0,00%	
Emissions	CO2	96,33	93,37	-3,07%
	SO2	321,7	315,92	-1,80%
	NOx	229,99	223,4	-2,87%
	Particles	22,22	21,53	-3,11%

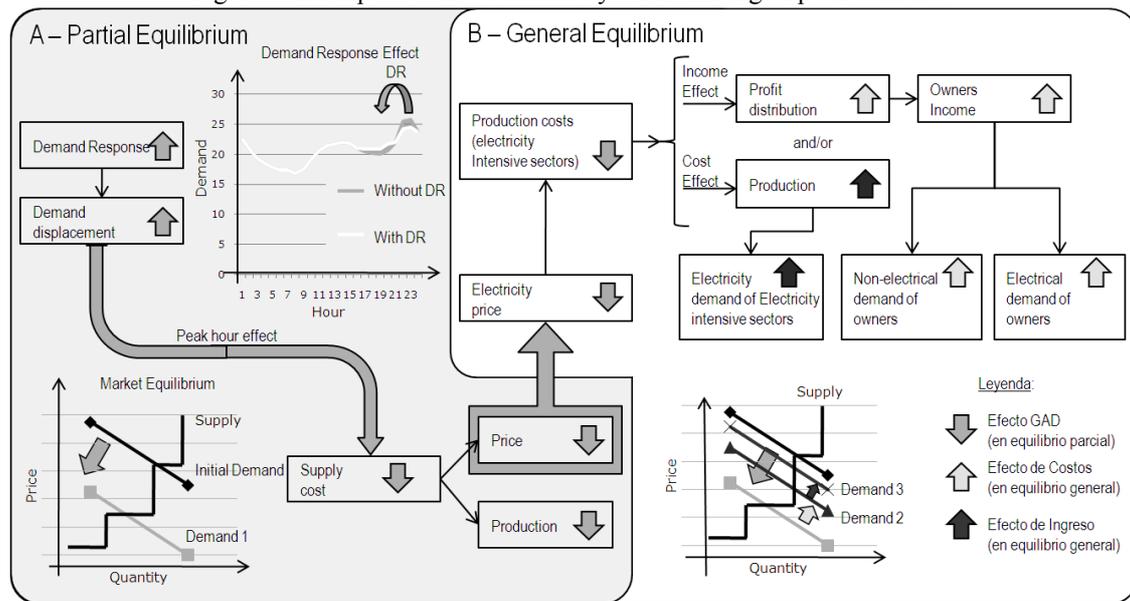
Source: Own elaboration based on Conchado & Linares (2009a) (2009b) works.

### III. General Equilibrium Model

As said before, the partial equilibrium model does an analysis of the DR effects taking into account only the productive structure of the electricity sector, analyzed under an assumption of an almost inelastic demand with a certain quantity of manageable demand for use in DR programs. As it is reasonable, the results shows that the introduction of DR produces a flattening of the system demand, reducing the peak consumption, the use of peak technologies as 'Gas' and 'Fuel Oil' and also, by counterpart, increasing the off-peak consumption. These changes alter the costs of delivering electricity for every load hourly level. However, the question to be answered is what happens when the effects of these changes on loads and prices are evaluated in other economic sectors?

One can argue that this question can only be answered by a general equilibrium model. Suppose for example the DR effect in the system for a specific load curve on a given day (Figure 6.A – Partial Equilibrium). As described before, the DR causes a change between the peak and off-peak consumption of the system (Figure 6.A – DR Effect). Considering the market equilibrium at a specific time point, the DR effect would cause a shift in demand to a lower level (Figure 6.A – Market Equilibrium). Taking into account the production costs of the electricity sector, this means that one can deliver the new demanded amount with cheaper production technologies, i.e. one can supply the demand with lower production costs bringing the possibility to the electricity price change to a lower level.

Figure 6. Example of effects caused by a DR shifting of peak demand.



Source: Own elaboration.

This is the result of the analysis considering only the direct effects of the power sector, in other terms; this is the result of the partial equilibrium analysis of the electricity sector. In order to assemble a truly overall analysis, it is necessary not only evaluate the power sector but also the various economic players that could suffer indirect effects caused by the DR changes.

Consequently, the partial modeling adopted provides the behavior of the electricity sector when facing the load displacement occurred because the DR programs. However, this simulation does not take into account important impacts that should be assessed as:

- the effective alteration of economic levels caused by the relationship of the electricity sector with other economic sectors;
- the indirect costs associated with changes in the cost of energy, both related with electricity and fuel;
- the effects on different economic actors decisions - other production sectors, households and government - and in the economic transactions with the rest of the world - imports and exports – of the implementation of a DR program;
- the environmental impact and the change in greenhouse gas emissions produced in economic sectors other than the electric.

For this reason, this paper utilizes a general equilibrium model to present the first results in the evaluation of indirect influences of changes promoted by DR programs, taking into account other (non-electric) productive sectors and demands of the Spanish economy.

To illustrate the difference of the approaches one could return to the example given before in Figure 6. The price change obtained due the opportunity of supplying the demand with cheaper technologies results for example, in reduced costs faced by the electricity-intensive sectors (Figure 6.B – General Equilibrium). That is to say, these sectors would have more money to invest in its own production or to guarantee simply more profits in their results.

If these sectors invest these savings from lower electricity costs in production, the output in this industry would rise, and therefore, as the sector in question is electricity intensive, the demand of electricity would also increase (Figure 6.B – Cost Effect). If the industry chooses not to invest their cost savings in production, these savings would be converted into profit distribution between its owners. In turn, these owners could use these additional revenues in the decision of increasing their own consumption of electricity, or more probably, in increasing the consumption of non-electric products that, on the other hand, make use of electricity as a production input (Figure 6.B – Income Effect).

The overall indirect effects present an opposite direction of the partial effects, smoothing the results previously obtained<sup>14</sup>. Meanwhile, the DR effects operate throughout the economy in a cyclical manner in the general equilibrium approach – the DR decreases the demand on the peak hours, which decreases the costs of supplying electricity, which lowers the electricity prices, which lowers the costs of non-electric sectors extending the benefits of these industries, which promotes the expansion of their own production or the distribution of additional revenues between their owners, which both effects increases the demand for electricity, which again alter the level of production of the electricity sector, finally leading to opposite effects of the initial DR consequence to electricity prices, starting the same cyclical process again.

Therefore, a general equilibrium model is a model that allows to endogenize these indirect effects in order to provide a more complete analysis of the effects of a policy on the economy of a country. The next section describes the computable general equilibrium model utilized in this work to measure these indirect effects of an increase of DR programs for the Spanish economy.

- **CGE model for Spanish economy**

The model used to simulate the Spanish economy is a Computable General Equilibrium Model (CGE) of neoclassical formulation, static, which models the relations of a country (Spain) with two outer regions (Europe and Rest of World), with the presence

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<sup>14</sup> This is true under the supposition of “well behaved” sector production functions, without the presence of reswitching of techniques or capital reversion, and in the presence of monotonous production functions.

of two production factors<sup>15</sup> (capital and labor), two institutions (government and representative household), twelve equivalent taxes based on Spanish system and 68 productive sectors (see Table 8 in Annex I – Extended Input-Output table).

A Social Accountability Matrix (SAM) has been used as a framework for consolidate the comprehensive economic data required to construct the CGE model. The SAM utilized represents simultaneously the macro-aggregates and input-output sectors information of the Spanish national accounts for the year 2000 and it is based on the Spanish national statistical database<sup>16</sup>.

The data requirements for the CGE formulation does not stop at the SAM. Historical elasticities of substitution between products and factors were obtained from economic databases such as GTAP (Global Trade Analysis Project). Meanwhile sectoral pollutant levels were drawn from national estimations also obtained from the Spanish national statistical database.

Following the structure described by Robinson (1989), in order to determine the CGE model, at first the economic agents should be specified. Second, one must specify behavioral rules that reflect the motivation of each agent. “For example, producers are typically assumed to maximize profits subject to technological constraints and households to maximize utility subject to income constraints.” (Robinson, 1989, p. 907). Third, the signals that influence the agents’ decision should be specified. In Walrasian’s CGEs, within the Arrow-Debreu (1954) tradition, the prices are the only signals that matter to agents. Fourth, the market structure should be specified to determine the institutional structure where agents interact. For example, perfect-competition implies agents as price-takers and flexible-prices.

In the Spanish CGE utilized in this paper the agents are represented by 68 productive sectors, a representative household, government and two external agents: Europe and the Rest of the World. The additional components of the CGE structure are briefly described below.

- **Productive sectors**

Each productive sector is described as a price-taker representative firm operating in a perfectly competitive market that chooses its production level by an analysis of production costs. The cost of producing each unit of output is determined by a production function that implies the possibility of substitution between inputs and productive factors. Their behavior objective is to maximize their profits, which under the above mentioned assumptions correspond to an equivalent cost minimization problem.

$$\text{Max:} \quad [Profit] = \left[ \begin{matrix} Firm \\ Income \end{matrix} \right] - \left[ \begin{matrix} Firm \\ Costs \end{matrix} \right] \quad \text{III.1}$$

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<sup>15</sup> The production factors usually correspond to services that could be sold or leased to firms by the households. In this model these are represented by the labor force and the capital ownership.

<sup>16</sup> Most of the Social Accountability Matrix (SAM) data can be obtained at the ‘Instituto Nacional de Estadística’ webpage (<http://www.ine.es>). Special thanks are due to Helena Vieitez and Miguel Rodriguez, from Vigo University, by providing assistance in the data acquiring and by constructing the SAM utilized in this model.

Subject to: [technological constraint] III.2

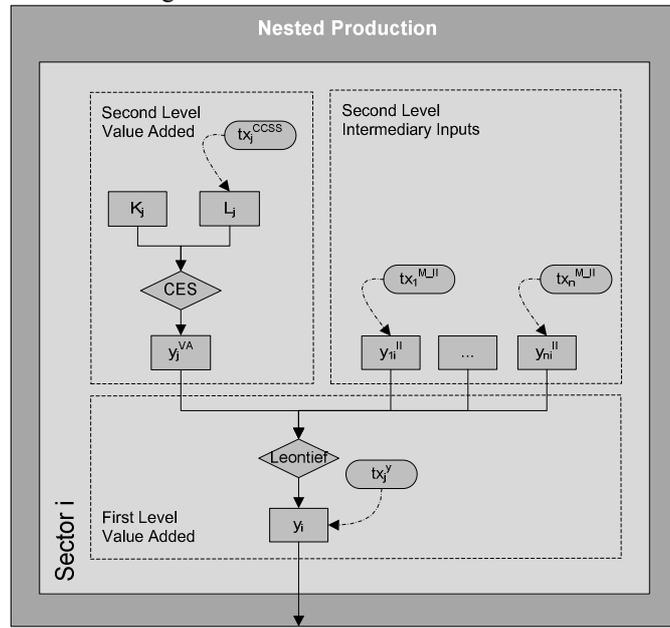
Max:  $\pi_j = p_i * y_j - C_j(p_i, w^f, y_j)$  III.3

Subject to:  $y_j = \varphi_j(y_{ij}^H, F_j^f)$  III.4

$y_j \geq 0$  III.5

The production process and technologies are represented in this model through a series of nested production functions (see Figure 7).

Figure 7. Nested Production functions.



Source: Own elaboration.

There are two levels of disaggregation. The second level is divided in two branches, one responsible for the formulation of a value added composite good and the other one for the definition of intermediate inputs quantities and prices used by the sector. For simplicity, we assume that the second level, the value added composite good ( $y_j^{VA}$ ), is represented only by the aggregation of two production factors types (Labor,  $L_j$ , and Capital,  $K_j$ ), combined through a constant elasticity of substitution function (CES)<sup>17</sup>.

The second level intermediate inputs produced in sector  $j$  and utilized by sector  $i$  ( $y_{ij}^{II}$ ) are described under a simpler approach. We consider that each intermediate input is bought at fixed proportions from the composite final goods sold to the market. Consequently the level of intermediate inputs used by each sector is directly obtained by its level of production ( $y_{ij}^{II} = c_{ij}^{II} y_j$ ), eliminating the need for additional calculations.

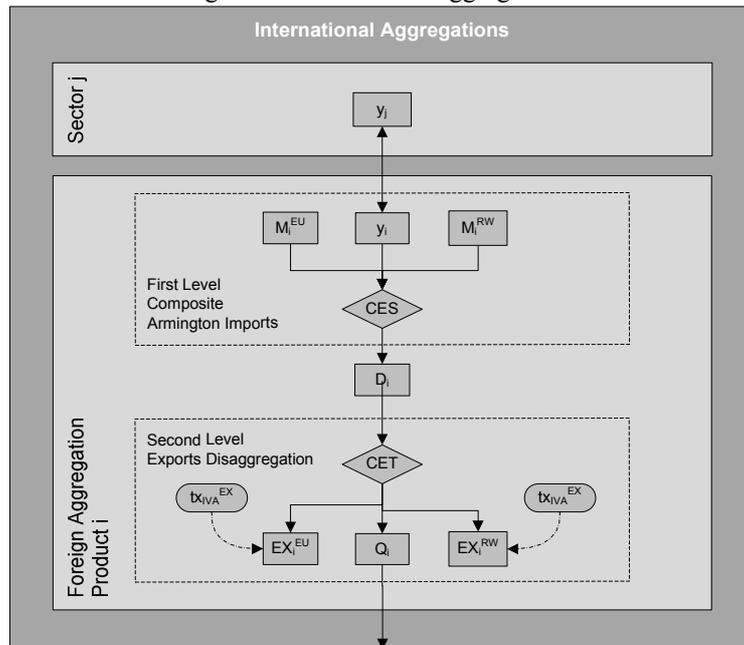
<sup>17</sup> A CES function has the form:  $Q = \left( \sum_{i=1}^n a_i^{\frac{1}{s}} * X_i^{\frac{(s-1)}{s}} \right)^{\frac{s}{s-1}}$ , where  $Q$  is the output,  $X_i$  for  $i = 1, \dots, n$  are the inputs used to produce  $Q$ ,  $a_i$  for  $i = 1, \dots, n$  are constants, and  $s$  is the elasticity of substitution.

After solving the second levels, it is possible to express correctly the demand and composite prices for each productive factor and intermediate input. Now, the first level production function is then representable. This level stands for the effective production decision of each sector, and its technology is represented by a Leontief<sup>18</sup> production function. The resulting Kuhn-Tucker conditions of the lower levels together with the solution of the first level are used to represent the optimal sector solution of production.

- **External sectors**

Once determined the domestic production decision, it is now necessary to deal with the transactions between the country and the exterior. The foreign demand and supply inclusions are made by incorporating the assumption of imperfect substitution between domestic production and goods and services imported and exported. Transactions between the country and abroad are shaped by the assumption of Armington elasticities of substitution (1969) for imports and a constant elasticity of transformation (CET) function for exports. It is also assumed that the country is not able to influence international prices, namely that Spain is price taker in the international markets.

Figure 8. International Aggregations.



Source: Own elaboration.

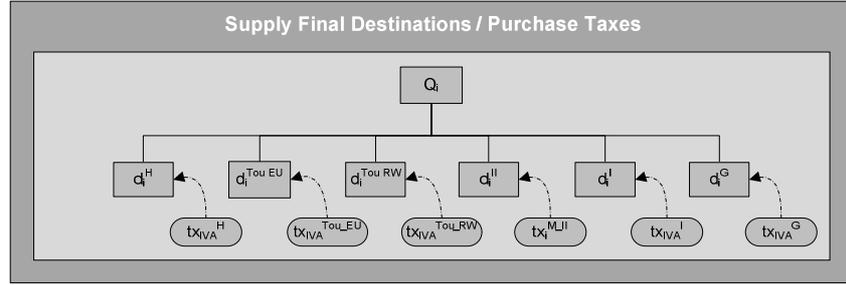
- **Final goods destination**

After the domestic production and external aggregations decisions have been determined, the resulting products offered in the domestic market can have six destinations. The total sales within the country ( $Q_i$ ) should be partitioned between the final consumption from households ( $d_i^H$ ), the consumption by foreign European tourists ( $d_i^{Tou-EU}$ ), the consumption by foreign tourists from the Rest of the World ( $d_i^{Tou-RW}$ ),

<sup>18</sup> The Leontief type production function is:  $Y = \min(x_i, x_j) = x_j$ , if  $x_j \geq x_i, j \neq i$ .

the intermediate consumption of goods  $i$  demanded by the productive sectors ( $d_i^{II}$ ), the investment goods demand ( $d_i^I$ ), and the public sector consumption demand ( $d_i^G$ ).

Figure 9. Final goods destination.



Source: Own elaboration.

The foreign tourist demand decision is taken into account through a simple representation. Their consumption decision depends of a fixed endowment of income in foreign currency, fully utilized in their expenditure decision through a fixed consumption share. The model also assumes that all savings are spent on investment goods, at fixed investment shares for each sector.

Assuming market clearing conditions for every product and factor of the economy, it is only necessary now to determine the household and government behavior to complete the description of the CGE model.

- **Private consumers**

Private consumers in the model share homothetic and identical preferences, and as consequence, they can be represented as a single representative household. Its objective is to choose its consumption bundle with the intention of maximizing its welfare, subject to a budget constraint. The welfare is represented by a utility function ( $U$ )<sup>19</sup> dependent on the final consumption of commodities in Spanish territory ( $d_i^H$ ), consumption in other countries ( $d_H^f$ ), and individual savings ( $S^H$ ), all subject to a budget constraint of a given level of income ( $Y^H$ ). The household income consists of earnings of the representative agent's endowment of production factors ( $\bar{L}^H$  and  $\bar{K}^H$ ) and transfers from the government and abroad ( $\bar{T}^{G-H}$  and  $\bar{T}^{f-H}$ ).

$$\text{Max:} \quad [\text{Welfare}] \quad \text{III.6}$$

$$\text{Subject to:} \quad [\text{budget constraint}] \rightarrow [\text{expenditures} \leq \text{Income}] \quad \text{III.7}$$

$$\text{Max:} \quad U(d_i^H, S^H) = \sum_{i=1}^n \bar{\mu}_i^H \cdot \ln d_i^H \quad \text{III.8}$$

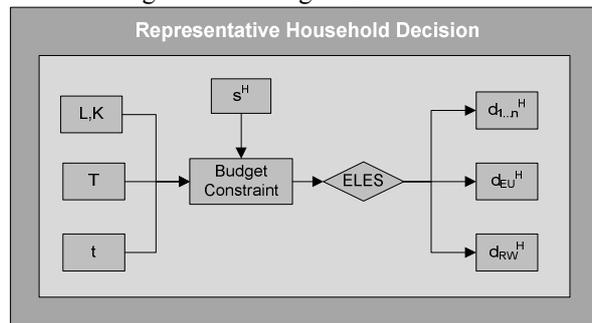
$$\text{Subject to:} \quad \left( \sum_{i=1}^n d_i^H * p_i^{Q-Htx} \right) + d_H^f \bar{\epsilon}^f \bar{p}_f^{\text{average}} \leq (1 - \bar{s}^H) Y^H \quad \text{III.9}$$

$$Y^H = p^L \bar{L}^H + p^K \bar{K}^H + \bar{T}^{G-H} + \bar{T}^{f-H} \quad \text{III.10}$$

<sup>19</sup> The household preferences are described in the shape of an extended linear expenditure system utility function.

$$d_i^H \geq 0$$

Figure 10. Final goods destination.



Source: Own elaboration.

- **Government**

The government has a simpler representation being considered a more stable agent. It receives income from its endowment of capital, tax and net foreign transfers. Its expenses are destined for its own public consumption, consisting of fixed proportions of their income, and net transfers to consumers. The deficit of governmental funding is represented by the variation of the public savings level.

## IV. Case Study Results

The assessment of the impact of DR programs in the Spanish economy is made through a comparative statics performed between the general equilibrium model results in a situation of absence of DR programs and its results under the insertion of a scenario of full penetration of electricity DR programs in the Spanish economy.

As referred before, increasing the DR causes the flattening of the system demand profile, reducing demand and production costs in peak hours. Regarding the Spanish economy, this result may be softened by indirect effects as the ones described in the beginning of Section III. The following sections present the results of the simulation of increasing the DR. However an important caveat should be explained before.

This CGE model as it is formulated is only able to analyze the reductions in electricity consumption caused by the DR programs, and not load shifting. Therefore, the results presented are due solely to load reductions through the utilization of equipments in more efficient modes, as ECO modes, and are not influenced by the displacement of load between peak and off-peak load levels. This limitation also implies that the influences of changes between fuel uses from changes in the electricity production technologies utilized are not analyzed, because the model is only capable of simulating fixed proportions reductions of the fuel use according to the reduction of electricity consumption. Research is under work to include the assessment of load shifting in the model.

Assuming this limitation, the results described below are the result of DR programs that produce domestic savings in electricity consumption corresponding to the maximal potential of penetration of DR programs, described in section II. The estimated level of

savings estimated corresponds to a decrease of 6.61% of the total household electricity consumption (Table 5) and their consequences are evaluated below.

#### IV. a. Analysis of demand response production impacts

Four Spanish structural factors are especially relevant in the analysis of CGE results. The first factor is derived directly from the partial equilibrium analysis, and corresponds to the determination of what production technologies will produce the electricity demanded. Its effect is the most straightforward of all of the effects examined. Sectors intimately linked to the electricity sector, producers in significant quantity of intermediate inputs for it, suffer a retraction on their production levels because of the shrinkage in electricity demand. Table 6 provides the list of sectors that might suffer more significantly this effect, such as fuels producers (coal, gas, crude oil, coke and refining).

Table 6. Intermediate Inputs used in the electricity production. Unit: Percentage.

	Share in the production of one unit of electricity
Intermediate Input	
Production and distribution of electricity	28,00%
Coke, refining and nuclear fuels	15,98%
Extraction of coal and lignite	14,90%
Production and distribution of gas	6,25%
Other business activities	6,21%
Sales and repair of vehicles and fuels	6,19%
Fabricated metal products	3,71%
Machinery and equipment	2,68%
Manufacture of electrical machinery and apparatus	2,52%
Others	13,56%

Source: Own elaboration.

The following three outlined factors are of more general influence and are related with the effects only evaluated under a general equilibrium structure. The first of these effects has the less indirect aspect: it is related to the use of electricity as an intermediate input of production in each specific sector, while the other two have an indirect influence character: they are related to each sector demand for production factors (capital and labor) necessary to obtain their final products.

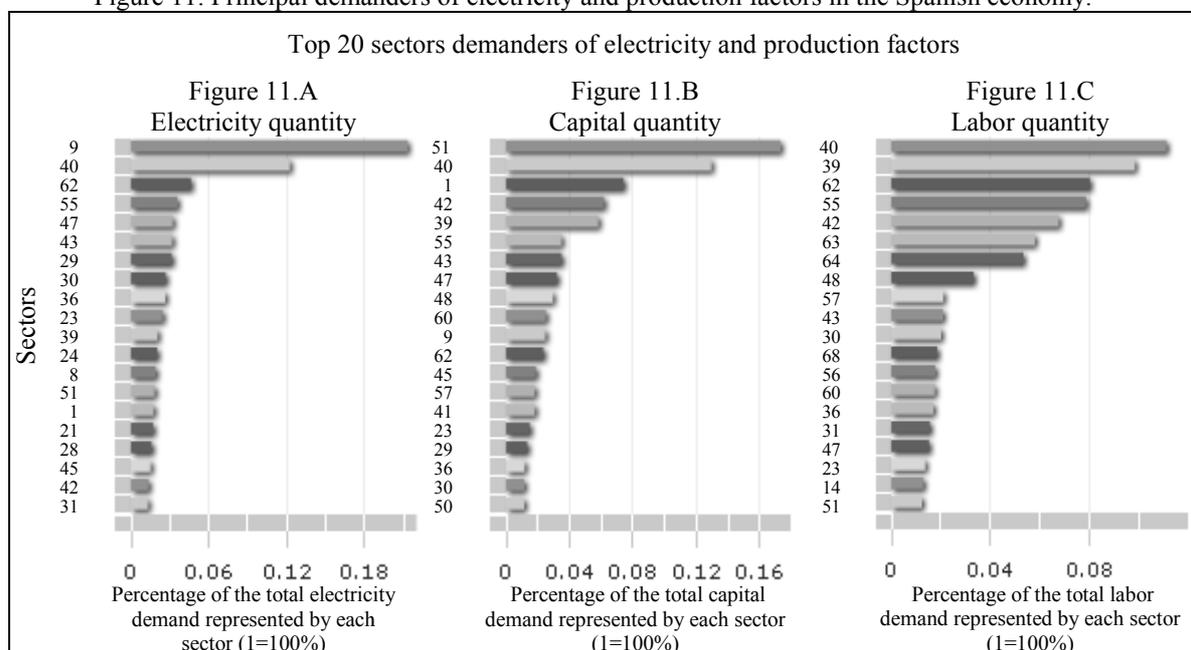
Two important dimensions when discussing these last three effects are: the total amount of electricity or production factor demanded by the industry (Figure 11) and the intensity with which the industry uses this factor (Figure 12). The sectors that use electricity in a significant amount per unit produced are called electricity intensive sectors (Figure 11.A), while sectors that use a more than proportional amount of a productive factor (capital or labor) in the production of one unit of product are called capital intensive (Figure 11.B) or labor-intensive sectors (Figure 11.C) respectively.

As noted in section II, a decrease in household electricity demand allows the possibility of supplying the quantity demanded by the system with cheaper production technologies, reducing the production costs and resulting in lower electricity prices. Evaluating as starting point the use of electricity as a productive input, the switch to a lower price level causes a drop in costs both in absolute and relative terms for sectors that use a significant amount of electricity in its production, creating more opportunities

for these sectors to convert their savings into growth of their own production or into the distribution of higher amounts of profit for its owners.

Figure 11.A describes the sectors most likely to be affected by this effect in absolute terms<sup>20</sup> while Figure 12.A describes the same in relative terms<sup>21</sup>.

Figure 11. Principal demanders of electricity and production factors in the Spanish economy.



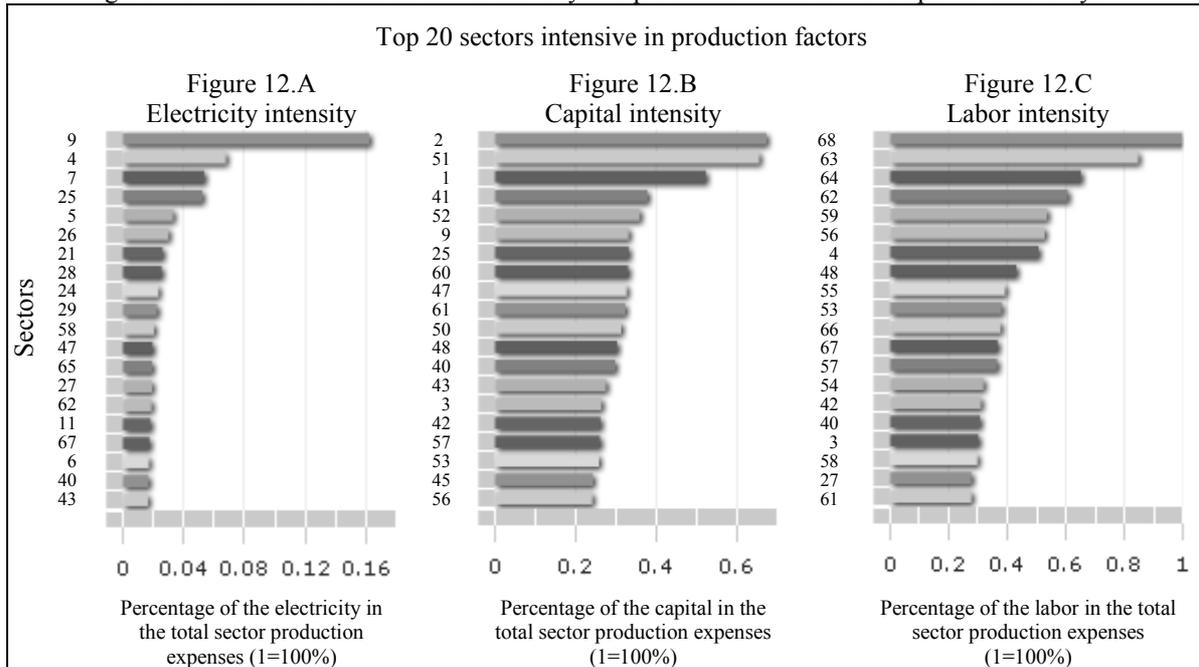
Source: Own elaboration. Unit: One ('1') corresponds to the total electricity or productive factors demanded by all productive sectors.

The last two traditionally analyzed sources of sectoral influence in CGE models are the demand of the sectors for the production factors labor and capital. To analyze the effect that DR programs can produce through this aspect in the activity level of each specific industry it is necessary firstly to analyze the structure of the electricity sector itself. As it is reasonable, and as it is shown in Figure 12.B, the electricity sector can be classified as a capital intensive industry (the sixth most intensive in the economy), and at the same time as one of the most capital demanding sector of the economy (is the tenth most capital demanding sector as can be seen in Figure 11.B).

<sup>20</sup> The most affected sectors by this change in absolute costs are: transportation-related services – sales and repair of vehicles and fuels sales (40), transport by rail, land and sea (43) and manufacture of motor vehicles and trailers (36) –, electricity-intensive services – public administration (62), other business activities (55), post and telecommunications –, traditional industrial sectors – metallurgy (29), fabricated metal products (30) and chemicals (23) – and construction (39).

<sup>21</sup> The sectors most affected in relative terms (electricity-intensive sectors) are: primary industries – coal mining (4), non-metallic minerals (7) and crude petroleum and natural gas (5) – and traditional industrial sectors – manufacturing of cement, lime and plaster (25), glass (26), paper (21), rubber and plastics (24), metallurgy (29) and other mineral products (28).

Figure 12. Most intensive sectors in electricity and production factors in the Spanish economy.



Source: Own elaboration. Unit: Percentage of electricity or production factor in the input expenses of the sector.

As a consequence, a decrease in the demand for electricity corresponds to a drop in the capital used by the power sector. As in the Spanish situation this sector is significantly important in relative terms (intensity) and in the absolute amount of the demanded capital of the whole economy, this drop in capital demand can create in turn significant effects. The shift in demand for capital lowers its price, which in turn benefits all other capital-intensive sectors and/or larger demanders of capital in the economy. Again, the savings generated by these sectors are reflected in increases of their own production or in a broader distribution of its profits to its owners.

Finally, the same mechanism can be used to evaluate the indirect effect which arises from the use of labor as productive factor. However, as can be seen in Figure 11.C and Figure 12.C, the electricity sector is neither intensive or great demander of labor, and therefore this last effect can be considered less significant in the analysis of the indirect effects of the implementation of DR programs.

The sum of the four previously described effects and the subsequent relationships between themselves and among external sectors (Europe and the Rest of the World) suggests the forces acting on the economy in the relocation of income, purchases and sales for each sector of the Spanish economy as a consequence of the introduction of DR programs. The production level resulting from this simulation can be described by the two figures presented below, both in absolute terms (Figure 13) or in relative terms (Figure 14).

Figure 13. Total sales difference (DR minus original levels) for each sold product of each sector.

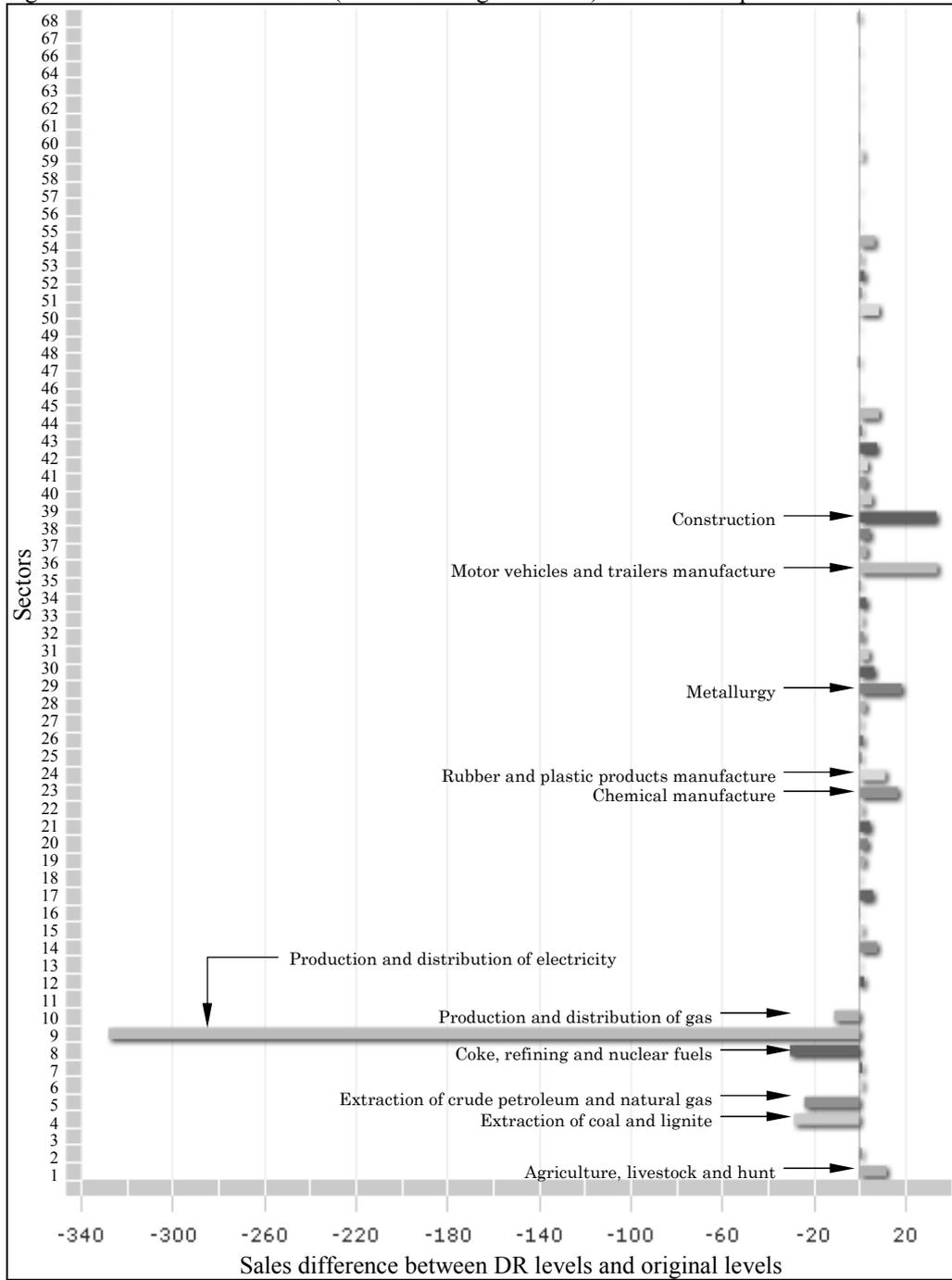
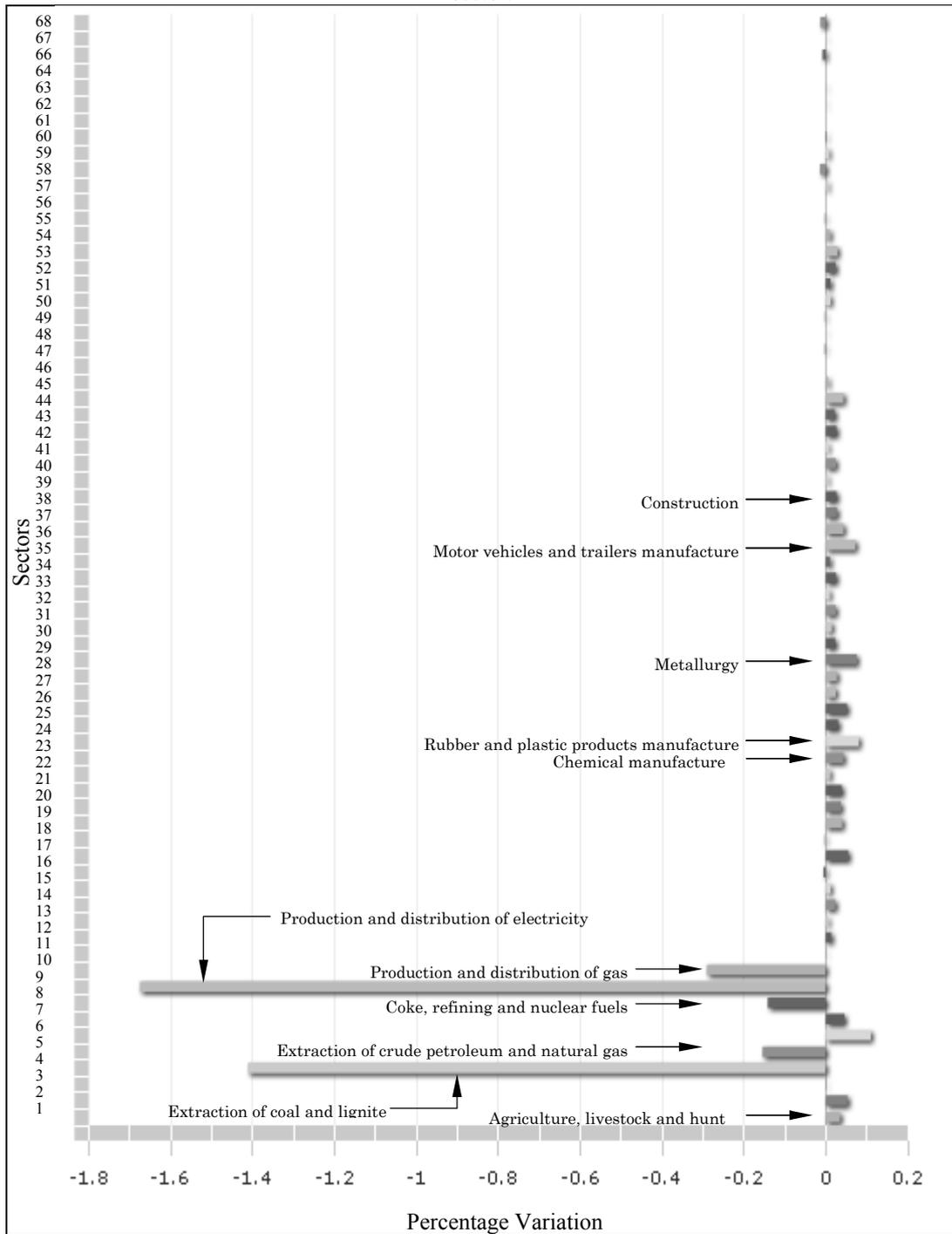


Figure 14. Percentage variation of total sales difference (quantity x prices) for each sold product of each sector.



Source: Own elaboration. Unit: percentage.

The four previously described effects have influence on the result obtained by the CGE model. However, only the first effect (from the partial equilibrium) has the same direction as the decrease in electricity production originated from the DR efficiency gains. All other effects contribute to lessening this effect on the influence over the activity level of each sector.

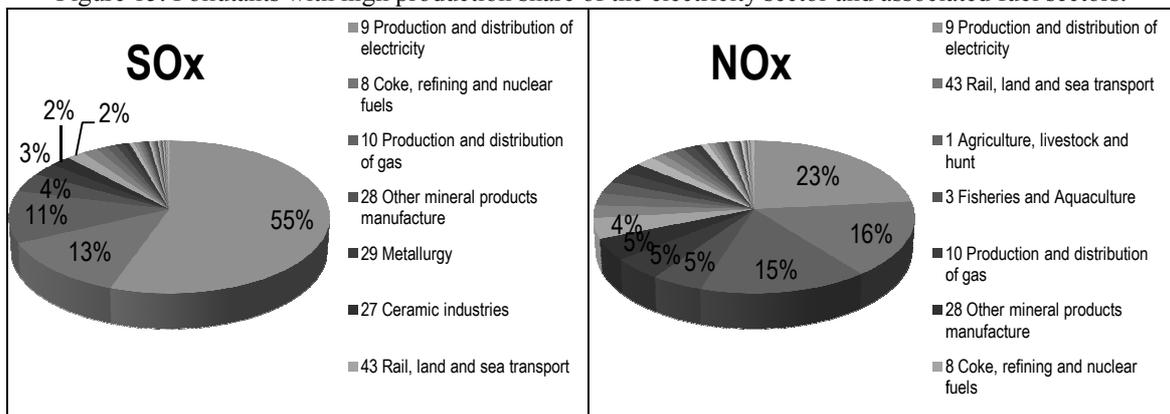
As expected, the sectors where the demand is closely linked to the level of production of the electricity sector have the first effect as the predominant one. Fuel suppliers for electricity are the sectors that are most impacted by the decline of electricity production. Coal mining (4), gas production (10), extraction of crude petroleum and natural gas (5) and coke, refining and nuclear fuels (8) are respectively the sectors that accompany the fall in production of the electricity sector. Here again, it is important to note the peculiarity that these effects reflect only proportional changes in the use of each technology in the original production of electricity, not reproducing any of the peak technology shifting usually promoted by a DR program.

Furthermore, even over these primarily affected sectors there is an influence of the three smoothing effects in the determination of their production levels. Nevertheless, it is in the other sectors that these indirect effects are more significant. The reduction of the purchase price of electricity and capital have strong repercussions enough to offset the drops in the electricity sector demands, promoting a increase in the activity level in sectors such as: manufacturing of motor vehicles and trailers (36), construction (39), metallurgy (29), agriculture, livestock and hunting (1), Chemical manufacturing (23), other food industries (14), other business activities (55), sale and repair of motor vehicles and fuels (40),...

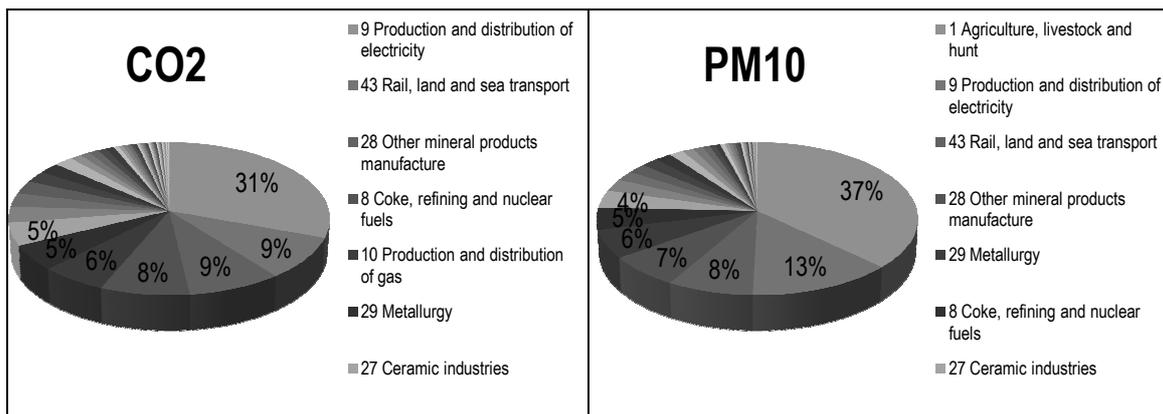
#### IV. b. Analysis of demand response emissions impacts

The changes on pollutants levels promoted by the DR program can be evaluated under two distinct groups. The first group relates to contaminants whose production is directly related to the electricity sector. This group is outlined in Figure 15, where the electricity sector together with the associated fuel producers are responsible for 80% of emissions of SO<sub>x</sub>, 33% of NO<sub>x</sub>, 45% CO<sub>2</sub> and 22% of particles in suspension with diameter up to 10 micrometers (TSP), according to 2000 data of the Spanish economy<sup>22</sup>.

Figure 15. Pollutants with high production share of the electricity sector and associated fuel sectors.



<sup>22</sup> Because of complications in acquiring data; most of the sector figures presented in this section refer to activities and pollutants levels of Spanish economy estimations in the year 2000. However, despite the fact that this data limits the direct comparison of the results obtained in the partial equilibrium model on absolute terms, the same does not occur when the comparison is made in relative terms. As the structural macroeconomic conjuncture and the productive sectors technology can be regarded as relatively stable for a period of few years, one can make the assumption that the figures obtained in this general equilibrium analysis are proxies for a simulation of DR programs implementation for the years around the first decade of the third millennium.



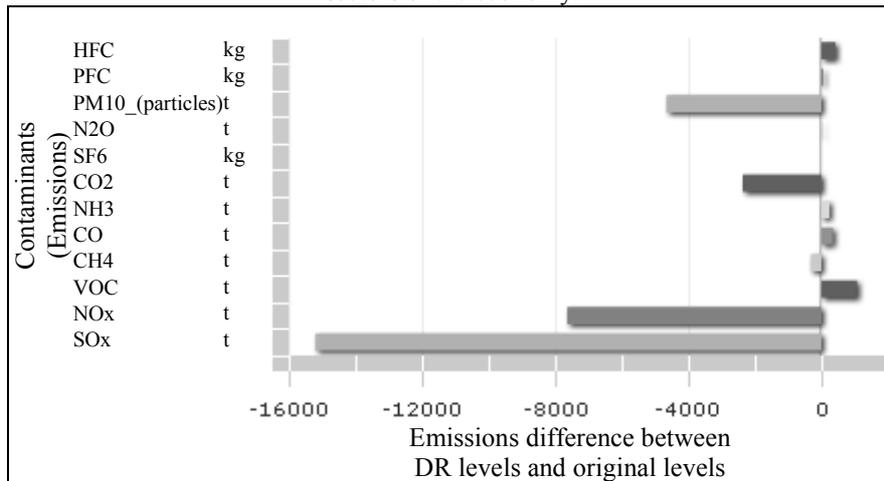
Source: Own elaboration. Unit: percentage.

In contrast, there is a second group of pollutants that can be outlined, which are related to the sectors not related to electricity production. Only 5% CH<sub>4</sub>, 4% CO, 2% of VOC, 2% N<sub>2</sub>O, and 0% other contaminants (NH<sub>3</sub>, SF<sub>6</sub>, PFCs and HFCs) are produced by the sectors directly related to electricity production.

As mentioned in the previous section, the major effects of shrinkage in the level of activity promoted by DR programs occurs in the electricity sector and in the sectors most intimately connected to it as suppliers of production inputs. Therefore, it is clear that the biggest changes in the amount of pollutants will occur in these sectors, and that they will accompany the fall in the level of electricity production. As can be noted in Figure 16 and Figure 17, the decreases in the level of emissions in the economy under a DR program correspond exactly to the pollutants listed in Figure 15. The changes on pollutants emissions are: -1.04% for SO<sub>x</sub> (-15,202 tonnes), -0.95% for CO<sub>2</sub> (-2,364 tonnes), -0.66% for NO<sub>x</sub> (-7,640 tonnes) and -2.91 for PM<sub>10</sub> (-4,651 tonnes).

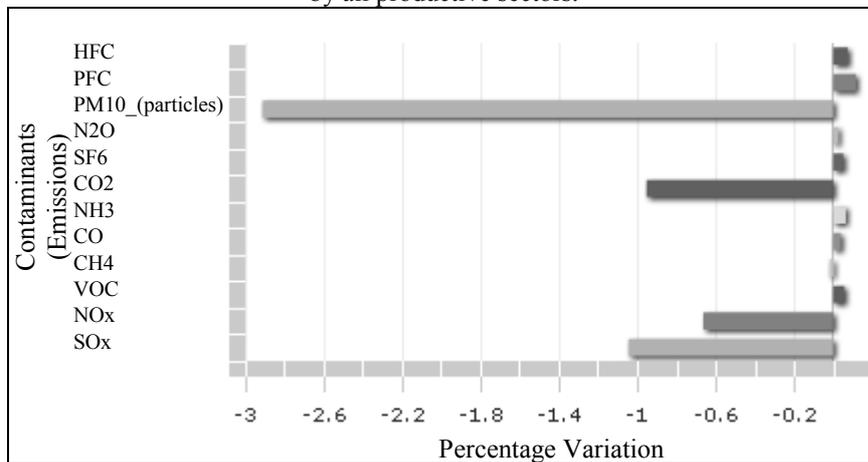
Pollutants belonging to the second group have their levels determined predominantly by the indirect effects of electricity, capital and labor price changes in the economy. The smaller change suffered by these sectors not directly related with the costs of production of electricity are translated to a smaller alteration of the pollutant levels under the new DR situation. More importantly, their effect tends to be in the opposite direction of the contaminants previously described, i.e., they tend to present an increase in emissions levels as a consequence of the expansion of the activity in these sectors. While CH<sub>4</sub> emissions still present a small influence of the electricity production levels corresponding to a low decrease of 0.01% of emissions (-325 tons), all other contaminants suffer an increase in the emissions level after the implementation of DR programs: N<sub>2</sub>O changes at 0.66% (19 tonnes), CO of 0.03% (346 tonnes), SF<sub>6</sub> of 0.05% (4 kg), VOC of 0.05% (1082 tonnes), NH<sub>3</sub> of 0.06% (240 tonnes), HFC 0, 07% (422 tonnes) and PFC of 0.11% (63 tonnes).

Figure 16. Difference of the quantity of pollutants emitted (DR minus original levels) for all productive sectors of the economy.



Source: Own elaboration. Unit: described in the graph.

Figure 17. Percentage variation of the quantity of pollutants emitted in the atmosphere with DR programs by all productive sectors.



Source: Own elaboration. Unit: percentage.

Furthermore, the comparison of the results obtained in the partial equilibrium and in the CGE simulation can show an indication of the benefits that a more comprehensive approach as the general equilibrium can promote when assessing pollutant emissions. Table 7 presents the relative results in pollutant emissions obtained by the partial and general equilibrium approaches for the implementation of a DR program.

Table 7. Results concerning the change in the emission levels of pollutants for the electricity sector (partial equilibrium) and for the whole economy (general equilibrium) models. Analysis applied to the Spanish economy in the scenario of complete penetration of DR programs.

	Partial equilibrium (only electricity sector emissions)	General equilibrium (total emissions in the economy)
Pollutant		
PM10_(particles)	-3,1%	-2,91%
SOx	-1,8%	-1,04%
CO2	-3,1%	-0,95%
NOx	-2,9%	-0,66%
CH4	-	-0,01%
N2O	-	0,02%
CO	-	0,03%
SF6	-	0,05%
VOC	-	0,05%
NH3	-	0,06%
HFC	-	0,07%
PFC	-	0,11%

Source: Own elaboration. Unit: Percentage. The absence of numbers ('-') means that the model does not calculate the emission levels.

The effect of the decrease of emissions obtained in the electricity sector is clearly maintained when evaluating the whole economy; however its spread to other sectors is declining, especially in the cases of CO<sub>2</sub> and NO<sub>x</sub>. In turn, DR programs promote, through the increase of activity of other sectors, an increase of emissions in smaller quantities of other atmospheric pollutants.

## V. Conclusions

This paper has used a CGE model of the Spanish economy to estimate the impact of implementing electricity DR programs in the economy. First it shows that there are two kinds of impacts that need to be taken in account in the evaluation of the DR policies: the direct and indirect effects. On one hand, the direct impacts address the expected changes in costs of the others industries induced by the changes in electricity production levels and prices. Concerning this impact it is expected that sectors with larger cross input/output interaction with the electricity generation experience a larger effect, as was underlined, in particular the cases of fuel industries as natural gas and coal.

On the other hand, the indirect effect appears from the consequences that electricity prices changes would have over other players' revenues and profits. The decrease on electricity costs may decrease other sectors costs as well as increase their production, consequently increasing their own electricity consumption and presenting substantial rebound effects.

The evaluation of the expected effects of DR policies is important in order to understand the different economic incentives promoted by the price signals. And these economic incentives do not concern only the electricity sector but also other economic sectors as a consequence of the matrix of inputs and outputs of the industries.

Demand response is seen by regulators as one of the main alternatives to face the problem of CO<sub>2</sub> emissions (OFGEM, 2009). And the model presented in this paper accomplishes an important step in the analysis of the policy impacts of such alternative

as it underlines the importance to include the economic interactions between different sectors. Additional studies need to be done in order to understand better the impact of electricity sector decisions on economic variables and to reduce the number of restrictive hypothesis, such as the assumption of proportional fuel decreases in relation to the electricity demand levels, which could be eliminated by representing the electricity production behavior in a more detailed way, internalizing in the model the different production technologies used for each load segment.

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## Annex I – Productive Sectors

Table 8. SAM productive sectors code.

	SAM – Year 2000
1	Agriculture, livestock and hunt
2	Forestry and logging
3	Fisheries and Aquaculture
4	Extraction of coal and lignite
5	Extraction of crude petroleum and natural gas. Extraction of uranium and thorium
6	Extraction of metallic minerals
7	Extraction of non-metallic mineral
8	Coke, refining and nuclear fuels
9	Production and distribution of electricity
10	Production and distribution of gas
11	Collection, purification and distribution of water
12	Meat manufacture
13	Milk manufacture
14	Other food industries
15	Beverages manufacture
16	Tobacco manufacture
17	Textile manufacture
18	Clothing and fur manufacture
19	Leather and footwear manufacture
20	Wood and cork manufacture
21	Paper manufacture
22	Publishing and printing
23	Chemical manufacture
24	Rubber and plastic products manufacture
25	Cement, lime and plaster manufacture
26	Glass and glass products manufacture
27	Ceramic industries
28	Other mineral products manufacture
29	Metallurgy
30	Metallic products manufacture
31	Machinery and equipment
32	Office machinery and computers
33	Electrical machinery and apparatus manufacture
34	Electronic material manufacture
35	Medical-surgical precision instruments
36	Motor vehicles and trailers manufacture
37	Other transport equipment manufacture
38	Furniture and other manufacturing industries. Recycling

39	Construction
40	Sales and repair of motor vehicles, motor fuel trade, Wholesale and intermediaries, retail trade, personal effects repair
41	Accommodation
42	Restoration
43	Rail, land and sea transport
44	Air and space transport
45	Auxiliary transport activities
46	Travel agencies activities
47	Post and telecommunications
48	Financial intermediation
49	Insurance and pension
50	Auxiliary activities
51	Estate activities. Imputed rent
52	Renting of machinery and household services
53	Computing activities
54	Search and development
55	Other business activities
56	Education of market
57	Health and social services of market
58	Public sanitation of market
59	Associative activities of market
60	Recreational, cultural and sports activities of market
61	Other personal services activities
62	Public administration
63	Non-market education
64	Non-market Health and social services
65	Non-market public sanitation from public administrations
66	Non-market associative activities from nonprofit institutions serving households
67	Non-market recreation and culture activities
68	Employed persons by households

## Annex II – Model Variables and Parameters

Parameters are differentiated from variables by a bar above the letter ( $\bar{a}$ ). Initial values are denoted by a 0 superscript (ex.  $p_i^0$ , means the initial price for good i).

### Variables:

#### Value Added Aggregation:

$y_j^{VA}$	Quantity of value added composite good produced by sector j
$p_j^{VA}$	Price of value added composite good of a specific sector j
$K_j$	Quantity of production factor capital utilized in a specific sector
$p^K$	Price of production factor Capital
$L_j$	Quantity of production factor labor utilized in a specific sector j
$p^L$	Price of production factor Labor (without social contributions taxes)
$p_j^{Ltx}$	Price of production factor Labor (with social contributions taxes)

#### Intermediate Inputs and Production Sector Aggregation:

$y_{ij}^I$	Quantity of intermediary input i utilized by a specific sector j
$y_j$	Quantity of the commodity produced by a specific sector j
$p_i$	Selling price of the commodity i (without foreign aggregations) (without production taxes)
$p_i^{ytx}$	Selling price of the commodity i (without foreign aggregations) (with production taxes)

#### Imports Aggregation:

$M_i^{EU}$	Final goods i imported from Europe
$M_i^{RW}$	Final goods i imported from the rest of the world
$D_i$	Final aggregated imported and domestic produced supply of a specific good i
$p_i^{M_{EU}}$	Price (in local currency) of imported goods i from Europe
$p_i^{M_{RW}}$	Price (in local currency) of imported goods i from the rest of the world
$p_i^D$	Final Armington aggregated price of the good produced by a specific sector i

#### Exports Disaggregation:

$EX_i^{EU}$	Final goods i exports to Europe
$EX_i^{RW}$	Final goods i exports to the rest of the world
$p_i^{EX_{EU}}$	Price (in local currency) of exported goods i to Europe (without exportation taxes)
$p_i^{EX_{RW}}$	Price (in local currency) of exported goods i to the rest of the world (without exportation taxes)
$p_i^{EX_{EUtx}}$	Price (in local currency) of exported goods i to Europe (with exportation taxes)

$p_i^{EX\_RWtx}$  Price (in local currency) of exported goods  $i$  to the rest of the world (with exportation taxes)

Final Goods:

$Q_i$  Final aggregated supply of a specific good  $i$  to domestic market

$p_i^Q$  Price of domestic supplied good  $i$

$p_i^{Q\_Htx}$  Household final purchase price (with taxes) of good  $i$  offered in the economy

$p_i^{Q\_Gtx}$  Government final purchase price (with taxes) of good  $i$  offered in the economy

$p_i^{Q\_Itx}$  Investment final purchase price (with taxes) of good  $i$  offered in the economy

$p_i^{Q\_Tou\_EUtx}$  European tourists final purchase price (with taxes) of good  $i$  offered in the economy

$p_i^{Q\_Tou\_RWtx}$  Rest of the world tourists final purchase price (with taxes) of good  $i$  offered in the economy

Destinations Balance:

$d_i^H$  Household domestic goods demand

$d_i^{Tou\_EU}$  Internal goods demand from European tourists

$d_i^{Tou\_RW}$  Internal goods demand from the rest of the world tourists

$d_i^{II}$  Intermediate inputs demand from productive sectors

$d_i^I$  Investment goods demand

$d_i^G$  Government goods demand

Capital and Labor market clearing:

$L^T$  Total demand of the production factor Labor

$K^T$  Total demand of the production factor Capital

Household Behaviour:

$d_H^{EU\_Tou}$  Household consumption abroad in Europe (household tourism in Europe)

$d_H^{RW\_Tou}$  Household consumption abroad in the rest of the world (household tourism in the rest of the world)

$Y^H$  Representative household income level

$E^H$  Household expenditure

Government Behaviour:

$Y^G$  Government income level

$E^G$  Government expenditure

Savings-Investment:

$S$	Total savings in the economy
$S^H$	Household total savings
$S^G$	Government total savings
$S^{EU}$	Europe total savings (in foreign currency)
$S^{RW}$	Rest of the world total savings (in foreign currency)
$I$	Total Investment in the economy

Numeraire:

$CPI$	Consumer Price Index
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Auxiliary Variable:

$WALRAS$	Walras check variable applied to saving investment equality
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**Parameters:**

Value Added Aggregation:

$\bar{\alpha}_j^{VA}$	Productivity parameter of sector value added composite good production function
$\bar{\alpha}_j^{VA,L}$	Share parameter of labor on value added composite good production function
$\bar{\alpha}_j^{VA,K}$	Share parameter of capital on value added composite good production function
$\bar{\sigma}_j^{VA}$	Elasticity of substitution between capital and labor productive factors

Intermediate Inputs and Production Sector Aggregation:

$\bar{c}_{ij}^{II}$	Share parameter of intermediate composites inputs utilized on sector production function
$\bar{c}_j^{VA}$	Share parameter of value added composite input utilized on sector production function

Imports Aggregation:

$\bar{\alpha}_i^D$	Productivity parameter of final aggregation supply good production function
$\bar{\alpha}_i^{D,y}$	Share parameter of domestic produced supply on production function
$\bar{\alpha}_i^{D,EU}$	Share parameter of European imports on production function
$\bar{\alpha}_i^{D,RW}$	Share parameter of rest of the world imports on production function
$\bar{\sigma}_i^D$	Elasticity of substitution between domestic-European-rest of the world offer goods
$\bar{\epsilon}^{EU}$	European exchange rate (direct quotation: 1 foreign currency unit = x home currency units)
$\bar{\epsilon}^{RW}$	Rest of the World exchange rate (direct quotation: 1 foreign currency unit = x home currency units)
$\bar{p}_i^{EU,M}$	International price of the imported goods from Europe

$\bar{p}_i^{RW\_M}$	International price of the imported goods from the rest of the world
$\bar{p}_i^{EU\_EX}$	International price of exported goods to Europe
$\bar{p}_i^{RW\_EX}$	International price of exported goods to the rest of the world

#### Exports Disaggregation:

$\bar{\beta}_i^{EX}$	Productivity parameter of sector products composite good transformation function
$\bar{b}_i^{EX\_Q}$	Share parameter of final aggregation supply good on transformation function
$\bar{b}_i^{EX\_EU}$	Share parameter of European exportation on transformation function
$\bar{b}_i^{EX\_RW}$	Share parameter of rest of the world exportation on transformation function
$\bar{\sigma}_i^{EX}$	Elasticity of transformation between domestic-European-rest of the world supply goods

#### Destinations Balance:

$\bar{\rho}_i^{Tou\_EU}$	European tourists fixed consumption share of national goods
$\bar{\rho}_i^{Tou\_RW}$	Rest of the World tourists fixed consumption share of national goods
$\bar{Y}^{Tou\_EU}$	Income of European tourists (in foreign currency)
$\bar{Y}^{Tou\_RW}$	Income of rest of the world tourists (in foreign currency)

#### Household Behaviour:

$\bar{L}^H$	Representative Household initial endowment of labor
$\bar{K}^H$	Representative Household initial endowment of capital
$\bar{s}^H$	Representative Household savings propensity
$\bar{\mu}_i^H$	Household marginal consumption propensity of a specific domestic good
$\bar{\mu}_{EU}^H$	Household abroad marginal consumption propensity in Europe
$\bar{\mu}_{RW}^H$	Household abroad marginal consumption propensity in the rest of the world
$\bar{p}_{EU}^{average}$	Average price index of European goods in foreign currency
$\bar{p}_{RW}^{average}$	Average price index of rest of the world goods in foreign currency

#### Government Behaviour:

$\bar{K}^G$	Government initial endowment of capital
$\bar{d}_i^{G\_initial}$	Government initial demand for goods

#### Transfers:

$\bar{T}^{G\_H}$	Transfers from Government to households
$\bar{T}^{H\_G}$	Transfers from households to Government
$\bar{T}^{EU\_H}$	Net transfers from Europe to households

$\bar{T}^{RW\_H}$	Net transfers from the rest of the world to households
$\bar{T}^{EU\_G}$	Net transfers from Europe to government
$\bar{T}^{RW\_G}$	Net transfers from the rest of the world to government

Savings-Investment:

$\bar{TK}^{EU}$	Net capital transfers from Europe in foreign currency
$\bar{TK}^{RW}$	Net capital transfers from the rest of the world in foreign currency
$\bar{\theta}_i$	Share parameter of demand for investment goods
$\bar{\theta}_{month,load\_block}$	

Taxes:

$\bar{tx}_j^{CCSSE}$	Employer social contributions tax rate
$\bar{tx}_j^{CCSSH}$	Employee's social contributions tax rate
$\bar{tx}_j^y$	Production tax rate
$\bar{tx}_j^{M\_II}$	Product tax over intermediate inputs sector goods purchases (import and specific taxes)
$\bar{tx}_{IVA}^H$	Product tax (IVA) over households purchases
$\bar{tx}_{IVA}^G$	Product tax (IVA) over government purchases
$\bar{tx}_{IVA}^I$	Product tax over gross capital formation
$\bar{tx}_{IVA}^{Tou\_EU}$	Product tax over European tourists purchases
$\bar{tx}_{IVA}^{Tou\_RW}$	Product tax over rest of the world tourists purchases
$\bar{tx}_{IVA}^{EX\_RW}$	European exportation product tax
$\bar{tx}_{IVA}^{EX\_EU}$	Rest of the world exportation product tax
$\bar{tx}^D$	Direct tax amount paid by households to government
$\bar{\mu}_{tx\_CCSSE}^{H\_G}$	Relation Household-Government payment for CCSSE social contributions
$\bar{\mu}_{tx\_CCSSH}^{H\_G}$	Relation Household-Government payment for CCSSH social contributions
$\bar{\mu}_{tx\_y}^{H\_G}$	Relation Household-Government payment for production taxes
$\bar{\mu}_{tx\_product}^{H\_G}$	Relation Household-Government payment for product taxes

Numeraire:

$\bar{\mu}_i^{CPI}$ and <i>elect</i>	Weight of the good on the consumer price index
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## Annex III – Model Equations

Type	Descriptions	Model Equations
Production Sector Equations – Value Added Aggregation	Value added production function by sector (CES)	$y_j^{VA} = \bar{a}_j^{VA} \left( \bar{a}_j^{VA,L} L_j \frac{\bar{\sigma}_j^{VA-1}}{\bar{\sigma}_j^{VA}} + \bar{a}_j^{VA,K} K_j \frac{\bar{\sigma}_j^{VA-1}}{\bar{\sigma}_j^{VA}} \right)^{\frac{\bar{\sigma}_j^{VA}}{\bar{\sigma}_j^{VA}-1}}$
	Labor capital transformation function	$\frac{L_j}{K_j} = \frac{(\bar{a}_j^{VA,L})^{\bar{\sigma}_j^{VA}} * (p^K)^{\bar{\sigma}_j^{VA}}}{(\bar{a}_j^{VA,K})^{\bar{\sigma}_j^{VA}} * (p^{L,tx})^{\bar{\sigma}_j^{VA}}}$
	Price of value added composite goods	$p_j^{VA} = \frac{p_j^{L,tx} * L_j + p^K * K_j}{y_j^{VA}}$
Production Sector Equations – Intermediate Inputs and Production Sector Aggregation	Demand inside the sector for value added composite goods to produce the sector output	$y_j = \frac{y_i^{VA}}{\bar{c}_i^{VA}}$
	Demand inside the sector for intermediary inputs to produce the sector output	$y_j = \frac{y_{1j}^I}{\bar{c}_{1j}^I} = \dots = \frac{y_{nj}^I}{\bar{c}_{nj}^I}$
	Unitary cost function to production on each sector	$p_j = \frac{p_j^{VA} * y_j^{VA} + (1 + \bar{t}x_j^{M,I}) * \sum_{i=1}^n p_i^Q * y_{ij}^I}{y_j}$
	Production price with taxes	$p_j^{y,tx} = p_j * (1 + \bar{t}x_j^y)$
International Accounts – Import Aggregation	Armington production function aggregation of imports and domestic produced goods (CES)	$D_j = \bar{a}_j^D \left( \bar{a}_j^{D,y} y_j \frac{\bar{\sigma}_j^D-1}{\bar{\sigma}_j^D} + \bar{a}_j^{D,EU} M_j^{EU} \frac{\bar{\sigma}_j^D-1}{\bar{\sigma}_j^D} + \bar{a}_j^{D,RW} M_j^{RW} \frac{\bar{\sigma}_j^D-1}{\bar{\sigma}_j^D} \right)^{\frac{\bar{\sigma}_j^D}{\bar{\sigma}_j^D-1}}$
	Domestic produced goods and imports from Europe transformation function	$\frac{y_j}{M_j^{EU}} = \frac{(\bar{a}_j^{D,y})^{\bar{\sigma}_j^D} * (p_j^{M,EU})^{\bar{\sigma}_j^D}}{(\bar{a}_j^{D,EU})^{\bar{\sigma}_j^D} * (p_j^{y,tx})^{\bar{\sigma}_j^D}}$
	Domestic produced goods and imports from RW transformation function	$\frac{y_j}{M_j^{RW}} = \frac{(\bar{a}_j^{D,y})^{\bar{\sigma}_j^D} * (p_j^{M,RW})^{\bar{\sigma}_j^D}}{(\bar{a}_j^{D,RW})^{\bar{\sigma}_j^D} * (p_j^{y,tx})^{\bar{\sigma}_j^D}}$
	Price of the CES aggregation (D) between domestic (y) and importation (M) goods supply	$p_j^D = \frac{p_j^{y,tx} * y_j + p_j^{M,EU} * M_j^{EU} + p_j^{M,RW} * M_j^{RW}}{D_j}$

International Accounts – Export Disaggregation	Transformation production function of exports and domestic Armington goods supply (CET)	$D_j = \bar{\beta}_j^{EX} \left( \bar{b}_j^{EX,Q} Q_j \frac{\bar{\sigma}_j^{EX+1}}{\bar{\sigma}_j^{EX}} + \bar{b}_j^{EX,EU} EX_j^{EU} \frac{\bar{\sigma}_j^{EX+1}}{\bar{\sigma}_j^{EX}} + \bar{b}_j^{EX,RW} EX_j^{RW} \frac{\bar{\sigma}_j^{EX+1}}{\bar{\sigma}_j^{EX}} \right)^{\frac{\bar{\sigma}_j^{EX}}{\bar{\sigma}_j^{EX}+1}}$
	Final goods supply and exports to Europe transformation function	$\frac{Q_j}{EX_j^{EU}} = \frac{(\bar{b}_j^{EX,RW})^{\bar{\sigma}_j^{EX}} * (p_j^Q)^{\bar{\sigma}_j^{EX}}}{(\bar{b}_j^{EX,Q})^{\bar{\sigma}_j^{EX}} * (p_j^{EX,RWtx})^{\bar{\sigma}_j^{EX}}}$
	Final goods supply and exports to RW transformation function	$\frac{Q_j}{EX_j^{EU}} = \frac{(\bar{b}_j^{EX,EU})^{\bar{\sigma}_j^{EX}} * (p_j^Q)^{\bar{\sigma}_j^{EX}}}{(\bar{b}_j^{EX,Q})^{\bar{\sigma}_j^{EX}} * (p_j^{EX,EUtx})^{\bar{\sigma}_j^{EX}}}$
	Price of the CET disaggregation of Armington aggregation (D) between exports (EX) and final goods (Q) supply	$p_j^Q = \frac{p_j^D * D_j + p_j^{EX,EU} * EX_j^{EU} + p_j^{EX,RW} * EX_j^{RW}}{Q_j}$
IVA prices	Final purchase price for households (with taxes) of the good offered in the economy	$p_i^{Q-Htx} = p_i^Q * (1 + \bar{t}x_i^{IVA-H})$
	Final purchase price for government (with taxes) of the good offered in the economy	$p_i^{Q-Gtx} = p_i^Q * (1 + \bar{t}x_i^{IVA-G})$
	Final purchase price for investment (with taxes) of the good offered in the economy	$p_i^{Q-Itx} = p_i^Q * (1 + \bar{t}x_i^{IVA-I})$
	Final purchase price for European tourists (with taxes) of the good offered in the economy	$p_i^{Q-Tou-EUtx} = p_i^Q * (1 + \bar{t}x_i^{IVA-Tou-EU})$
	Final purchase price for rest of the world tourists (with taxes) of the good offered in the economy	$p_i^{Q-Tou-RWtx} = p_i^Q * (1 + \bar{t}x_i^{IVA-Tou-RW})$
Destinations Balance Equations	Balance equation of possible final goods destinations	$Q_i = d_i^H + d_i^{Tou-EU} + d_i^{Tou-RW} + d_i^I + d_i^J + d_i^G$
	Balance equation of intermediate inputs sales and demand	$d_i^I = \sum_{j=1}^n y_{ij}^I$
	European tourists demand for goods in national territory	$d_i^{Tou-EU} = \frac{\bar{\rho}_i^{Tou-EU} * \bar{\epsilon}^{EU} * \bar{\gamma}^{Tou-EU}}{p_i^{Q-Tou-EUtx}}$

	Rest of the world tourists demand for goods in national territory	$d_i^{Tou\_RW} = \frac{\bar{\rho}_i^{Tou\_RW} * \bar{\epsilon}^{RW} * \bar{Y}^{Tou\_RW}}{p_i^{Q\_Tou\_RWtx}}$
Household Equations	Household income	$Y^H = p^L * \bar{L}^H + p^K * \bar{K}^H + \bar{T}^{G\_H} + \bar{T}^{EU\_H} + \bar{T}^{RW\_H} + \left[ \bar{\mu}_{tx\_CCSSE}^{H\_G} * \sum_{j=1}^n \bar{tx}_j^{CCSSE} * L_j * p^L \right]$ $+ \left[ \bar{\mu}_{tx\_CCSSH}^{H\_G} * \sum_{j=1}^n \bar{tx}_j^{CCSSH} * L_j * p^L \right] + \left[ \bar{\mu}_{tx\_y}^{H\_G} * \sum_{j=1}^n \bar{tx}_j^y * y_j * p_j \right]$ $+ \left\{ \bar{\mu}_{tx\_product}^{H\_G} * \left[ \sum_{j=1}^n \left( \sum_{i=1}^n p_i^Q * y_{ij}^H * tx_j^{M,II} \right) \right] \right\}$ $+ \left[ \sum_{i=1}^n \bar{tx}_{IVA}^{EX\_EU} * EX_i^{EU} * p_i^{EX\_EU} \right] + \left[ \sum_{i=1}^n \bar{tx}_{IVA}^{EX\_RW} * EX_i^{EU} * p_i^{EX\_EU} \right]$ $+ \left[ \sum_{i=1}^n \bar{tx}_{IVA}^H * d_i^H * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{tx}_{IVA}^G * d_i^G * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{tx}_{IVA}^I * d_i^I * p_i^Q \right]$ $+ \left[ \sum_{i=1}^n \bar{tx}_{IVA}^{Tou\_EU} * d_i^{Tou\_EU} * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{tx}_{IVA}^{Tou\_RW} * d_i^{Tou\_RW} * p_i^Q \right] \left. \right\}$
	Household domestic goods demand	$d_i^H = \frac{\bar{\mu}_i^H * [(1 - \bar{s}^H) * (Y^H - \bar{T}^{H\_G} - \bar{tx}^D)]}{p_i^{Q\_Htx}}$
	Household consumption abroad in Europe (household tourism in Europe)	$d_H^{EU\_Tou} = \frac{\bar{\mu}_{EU}^H * [(1 - \bar{s}^H) * (Y^H - \bar{T}^{H\_G} - \bar{tx}^D)]}{\bar{\epsilon}^{EU} \bar{p}_{EU}^{average}}$
	Household consumption abroad in the rest of the world (household tourism in the rest of the world)	$d_H^{RW\_Tou} = \frac{\bar{\mu}_{RW}^H * [(1 - \bar{s}^H) * (Y^H - \bar{T}^{H\_G} - \bar{tx}^D)]}{\bar{\epsilon}^{RW} \bar{p}_{RW}^{average}}$
	Household Expenditure	$E^H = \left( \sum_{i=1}^n d_i^H * p_i^{Q\_Htx} \right) + d_H^{EU\_Tou} \bar{\epsilon}^{EU} \bar{p}_{EU}^{average} + d_H^{RW\_Tou} \bar{\epsilon}^{RW} \bar{p}_{RW}^{average} + \bar{T}^{H\_G} + \bar{tx}^D$
	Household total savings	$S^H = \bar{s}^H (Y^H - \bar{T}^{H\_G} - \bar{tx}^D)$

Government Equations	Government income	$ \begin{aligned} Y^G = & p^K * \bar{K}^G + \bar{T}^{H,G} + \bar{T}^{EU,G} + \bar{T}^{RW,G} + \bar{t}x^D + \left[ (1 - \bar{\mu}_{tx\_CCSSE}^{H,G}) * \sum_{j=1}^n \bar{t}x_j^{CCSSE} * L_j * p^L \right] \\ & + \left[ (1 - \bar{\mu}_{tx\_CCSSH}^{H,G}) * \sum_{j=1}^n \bar{t}x_j^{CCSSH} * L_j * p^L \right] \\ & + \left[ (1 - \bar{\mu}_{tx\_y}^{H,G}) * \sum_{j=1}^n \bar{t}x_j^y * y_j * p_j \right] \\ & + \left\{ (1 - \bar{\mu}_{tx\_product}^{H,G}) * \left[ \sum_{j=1}^n \left( \sum_{i=1}^n p_i^Q * y_{ij}^H * t x_j^{M,II} \right) \right] \right\} \\ & + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^{EX,EU} * EX_i^{EU} * p_i^{EX,EU} \right] + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^{EX,RW} * EX_i^{EU} * p_i^{EX,EU} \right] \\ & + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^H * d_i^H * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^G * d_i^G * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^I * d_i^I * p_i^Q \right] \\ & + \left. \left[ \sum_{i=1}^n \bar{t}x_{IVA}^{Tou,EU} * d_i^{Tou,EU} * p_i^Q \right] + \left[ \sum_{i=1}^n \bar{t}x_{IVA}^{Tou,RW} * d_i^{Tou,RW} * p_i^Q \right] \right\} \end{aligned} $
	Government expenditure	$E^G = \left( \sum_{i=1}^n d_i^G * p_i^{Q,Gtx} \right) + \bar{T}^{G,H}$
	Government domestic goods demand	$d_i^G = \bar{d}_i^{G,initial}$
	Government total savings	$S^G = Y^G - E^G$
Labor and Capital Market Clearing	Price of production factor Labor (with social contributions taxes)	$p_j^{Ltx} = p^L * (1 + \bar{t}x_j^{CCSSE} + \bar{t}x_j^{CCSSH})$
	Labor total quantity	$L^T = \sum_{j=1}^n L_j$
	Capital total quantity	$K^T = \sum_{j=1}^n K_j$
	Labor market clearing	$\bar{L}^H = L^T$
	Capital market clearing	$\bar{K}^H + \bar{K}^G = K^T$

Savings-Investment Equations	Total savings in the economy	$S = S^H + S^G + \bar{\epsilon}^{EU} S^{EU} + \bar{\epsilon}^{RW} S^{RW} + \bar{\epsilon}^{EU} \bar{T}K^{EU} + \bar{\epsilon}^{RW} \bar{T}K^{RW}$
	Europe total savings (in foreign currency)	$S^{EU} = \left[ \left( \sum_{i=1}^n \frac{p_i^{M,EU} M_i^{EU}}{\bar{\epsilon}^{EU}} \right) + (d_H^{EU, Tou} \bar{p}_{EU}^{average}) \right] - \left[ \left( \sum_{i=1}^n \frac{p_i^{EX,EU,tx} EX_i^{EU}}{\bar{\epsilon}^{EU}} \right) + \left( \sum_{i=1}^n \frac{p_i^{Q, Tou, EU, tx} d_i^{Tou, EU}}{\bar{\epsilon}^{EU}} \right) + \frac{\bar{T}^{EU, H}}{\bar{\epsilon}^{EU}} + \frac{\bar{T}^{EU, G}}{\bar{\epsilon}^{EU}} + \bar{T}K^{EU} \right]$
	Rest of the world total savings (in foreign currency)	$S^{RW} = \left[ \left( \sum_{i=1}^n \frac{p_i^{M, RW} M_i^{RW}}{\bar{\epsilon}^{RW}} \right) + (d_H^{RW, Tou} \bar{p}_{RW}^{average}) \right] - \left[ \left( \sum_{i=1}^n \frac{p_i^{EX, RW, tx} EX_i^{RW}}{\bar{\epsilon}^{RW}} \right) + \left( \sum_{i=1}^n \frac{p_i^{Q, Tou, RW, tx} d_i^{Tou, RW}}{\bar{\epsilon}^{RW}} \right) + \frac{\bar{T}^{RW, H}}{\bar{\epsilon}^{RW}} + \frac{\bar{T}^{RW, G}}{\bar{\epsilon}^{RW}} + \bar{T}K^{RW} \right]$
	Investment demand for each good	$d_i^I = \frac{\bar{\theta}_i * I}{p_i^{Q, I, tx}}$
	Savings equals to investment	$I = S + WALRAS$
Additional Price Equations	Final goods exportation to Europe prices (in local currency)	$p_i^{EX, EU} = \bar{\epsilon}^{EU} * \bar{p}_i^{EU, EX}$
	Final goods exportation to the rest of the world prices (in local currency)	$p_i^{EX, RW} = \bar{\epsilon}^{RW} * \bar{p}_i^{RW, EX}$
	Final goods exportation to Europe prices with taxes (in local currency)	$p_i^{EX, EU, tx} = p_i^{EX, EU} * (1 + \bar{t}x_{IVA}^{EX, EU})$
	Final goods exportation to the rest of the world prices with taxes (in local currency)	$p_i^{EX, RW, tx} = p_i^{EX, RW} * (1 + \bar{t}x_{IVA}^{EX, RW})$
	Final goods importation from Europe prices (in local currency)	$p_i^{M, EU} = \bar{\epsilon}^{EU} * \bar{p}_i^{EU, M}$
	Final goods importation from Europe prices (in local currency)	$p_i^{M, RW} = \bar{\epsilon}^{RW} * \bar{p}_i^{RW, M}$

	Consumer price index	$CPI = \sum_{i=1}^n (\bar{\mu}_i^{CPI} * p_i^Q)$

## **4. RESEARCH WORK III**

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### **INTRODUCTION**

This section contains the research work III entitled:  
IMPROVING THE REPRESENTATION OF THE ELECTRICITY SECTOR IN  
COMPUTABLE GENERAL EQUILIBRIUM MODELS



# IMPROVING THE REPRESENTATION OF THE ELECTRICITY SECTOR IN COMPUTABLE GENERAL EQUILIBRIUM MODELS

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## **Abstract:**

Energy policy evaluation modelers traditionally have been divided between two disjoint modeling approaches due to data and other incompatibilities: bottom-up and top-down models. Some efforts have been attempted to incorporate technological richness in top-down models, especially by the introduction of supply-side electricity generation detail into accountability frameworks, however electricity demand disaggregation has been completely neglected in general equilibrium models. This paper develops a solution to represent both supply technological richness and demand time heterogeneous electricity behavior in a way that is consistent with a social accounting framework. Goal programming approach was utilized in order to achieve the numerical calibration of the significant amount of economic and engineering data required. Moreover, physical constraints as thermal efficiency transformation of fuels were dealt in order to avoid inconsistencies on the economical representation and time based disaggregation presented. The process is illustrated with data from Spanish electricity sector.

**Keywords:** Energy modeling, Electricity power technologies and demand blocks, Computable General Equilibrium (CGE), Social Accountability matrix (SAM).

**JEL classification:** C68, D58, Q4, Q51, L60

## **I. Introduction**

The growing relevance of energy and climate issues is posing increasing challenges on public policy design and modeling. Indeed, the design and assessment of public policies in these fields requires more detailed approaches as energy systems become more complex, with a wider choice of technologies and demand alternatives. Additionally, the large implications of energy choices on the economy and climate consequences also ask for a wide-ranging, encompassing approach of all these issues.

In particular, the increasing electrification of energy systems across the world (partly as a response to climate concerns) requires an accurate representation of the electricity sector, if a proper policy assessment is to be achieved. However, up to now, this accurate representation has been incompatible with a proper representation of the economy and the environment, mostly due to two reasons: computational requirements, and data compatibility.

This has resulted in two disjoint modeling approaches: bottom-up (BU) models and top-down (TD) ones. BU models are able to represent a detailed electricity or energy sector, including different production technologies, demand levels, and technical and network constraints. However, they are not able to represent correctly the linkage of this sector with the rest of the economy. On the other hand, TD models, which are able to represent this linkage, only represent electricity as an aggregated commodity, produced only one time each period by a combination of diverse production factors and other commodities.

Much research has been devoted lately to the reconciliation of these two modeling approaches, or hybrid modeling (Böhringer, 1998; Böhringer & Loschel, 2006; Labandeira, Linares, & Rodríguez, 2009; Turton, 2008; Wene, 1996). Hybrid models include a TD structure and in unison incorporate a detailed production description of the electricity sector, as complete as possible.

However, although some intelligent proposals have been made to solve the computational requirements, the data compatibility issue still remains. Koopmans & Velde, (2001); Mcfarland (2004); Mcfarland & Herzog (2006); Ghersi & Hourcade (2006); Sue Wing (2006) and Sue Wing (2008) are all examples of different works in pursuit of consistency in the incorporation of technological richness in TD models by adapting the databases used. Sue Wing (2008) addressed this issue by determining with a fairly degree of accuracy a procedure to approximate the compatibility of technological data insertion in the supply side of a TD problem. The electricity sector activities (Generation (G), Transmission and Distribution (T&D)) and the electricity generation technological details were harmonized under a Social Accountability framework.

As mentioned before, all these efforts have been directed to the incorporation of the technological richness of the electricity sector into the models. But there is another significant feature of electricity production which, to the authors' knowledge, has not been properly incorporated yet into hybrid models, namely the non-storability of electricity. This non-storability makes electricity in fact a heterogeneous commodity, with different combinations of production factors for each demand level, and also with different production prices because of this dependence on time. Therefore, the current representation of electricity as a single homogeneous product can be considered a very strong simplification in energy model. This is even more important in liberalized electricity markets with marginal settling prices, which can diverge significantly from the single-period average price used in TD models.

The detailed representation of time blocks or demand levels on TD models is the major contribution of this paper. Therefore, this paper introduces a TD detailed procedure addressing the integration of not only the technological production richness of bottom-up data, but also of a load block electricity demand representation into a social accountability framework. This formulation is not only important for potentially widening general equilibrium models applications but also in allowing to minimize the inherent data and variables incompatibilities problems on hybrid modeling representations.

Two additional contributions can be underlined in this paper. Firstly, the problem of representing correctly thermal efficiency transformation of fuels on economical models is described and adapted from previous studies to this paper case. Secondly, the data reconciliation problem is formulated on a more intuitive, linear way, which improves and simplifies the estimations proposed by the existing literature.

The application of such extension is illustrated with data from Spanish electricity sector. And examples of policy simulations impossible to attain under a supply-side-only top-down approach, as effects of electricity load displacement in demand response programs, are provided to outline the strength of the detailed treatment of heterogeneous electricity demand in TD models.

## II. Accountability framework: Embedded restrictions

Any TD model requires a data structure capable of conferring a consistent picture of the economy at a specific moment in time. National accounts and social accountability matrices (SAM) are the common point of departure for this, as can be seen in the simplified SAM structure presented in Table 9. In this case, the national economy transactions are represented by two goods, Energy (E) and additional activities (Q), used as intermediate inputs resources (II) by sectors (Q and E) in unison with Labor (L) and Capital (K) production factors to produce the final economic goods consumed by Households (H) or used in the Gross Fixed Capital Formation (GFCF).

Table 9. Schematic social accountability.

		Uses			
		Q	E	H	GFCF
Resources	Q	$\overline{P_Q II_{QQ}}$	$\overline{P_Q II_{QE}}$	$\overline{P_Q Q_H}$	$\overline{P_Q Q_k}$
	E	$\overline{P_E II_{EQ}}$	$\overline{P_E II_{EE}}$	$\overline{P_E E_H}$	-
	L	$\overline{wL_Q}$	$\overline{wL_E}$		
	K	$\overline{rK_Q}$	$\overline{rK_E}$		

Source: Own elaboration based on (Ghersi & Hourcade, 2006).

On the other hand, BU electricity models describe the electricity sector as a set of discrete technologies (or units of production) that are characterized by a specific fuel-use efficiency and different investment, operational and maintenance costs. Evidently, the common variables between both data structures are the ones direct related with the electricity sector production. In terms of quantity, the total electricity produced (E) and the household electricity consumption ( $E_H$ ) are the main points of intersection. In terms of prices, BU models deal with costs and marginal values of technologies while national accounts deal with a simpler aggregation by using average prices of electricity.

As an alternative approximation between these two disjoint structures, Sue Wing (2008) developed a method to numerically calibrate the economic and engineering data by positive mathematical programming. Focusing in translating the electricity sector activities structure: transmission, distribution and generation (and further its generation technologies), the supply structure of the sector was decomposed in order to produce a compatible TD structure, as in the scheme represented in Table 10.

Table 10. Schematic social accountability with supply-side electricity disaggregation.

		Uses					
		Q	T&D	Generation		H	GFCF
				tech 1	... tech n		
Resources	Q	$\overline{P_Q I_{QQ}}$	$P_Q I_{QT\&D}$	$P_Q I_{QG_{tech\ i}}$		$\overline{P_Q Q_H}$	$\overline{P_Q Q_k}$
	E	$\overline{P_E I_{EQ}}$	$P_E I_{ET\&D}$	$P_E I_{EG_{tech\ i}}$		$\overline{P_E E_H}$	-
	L	$\overline{wL_Q}$	$wL_{T\&D}$	$wL_{G_{tech\ i}}$			
	K	$\overline{rK_Q}$	$rK_{T\&D}$	$rK_{G_{tech\ i}}$			

Source: Own elaboration.

The electricity sector (and more specifically the resources used in its production structure) is converted into a series of extra columns in order to embrace the new activities subdivision. The new structure obtained is capable of addressing issues related with the technological structure of the electricity sector; nevertheless, this data arrangement does not solve entirely the convergence problem between the two disjoint modeling approaches.

The information contained in average electricity prices is not able to truthfully reflect the actual behavior of electricity prices in competitive, marginal-price electricity markets. In these markets, the electricity generation price corresponds to the bid of the marginal unit -- the last needed power plant to be dispatched at each time period --, and has no direct relation with average prices.

Therefore, there is no guarantee that an increase in the electricity demand would present an additional cost in the neighborhood of the average cost reflected in the national accounts. Actually, even the direction of the effect in prices is uncertain without further information. For example, an increase in the electricity demand in hours of lower demand levels (base periods) would present a cost lower than the average price of electricity, since the additional energy needed to be produced could make use of cheaper variable cost power plants. As a consequence, the increase in demand would actually decrease the average price of electricity in the national accounts. Meanwhile, the opposite effect would occur if the increase in demand happens in peak hours of the day, because costs incurred by the need of using more expensive variable cost units of production to serve the new demand would be greater than the initial average electricity price.

It is then evident that in any policy evaluation where electricity demand is considered, it is important to regard electricity as a heterogeneous commodity. Therefore, an extension to the former disaggregation proposition is necessary.

One of the alternatives to reconcile the TD database with BU representations, considering this limitation, would be taking into account the existence of a set of different electricity products. An extension of the previously described 'supply-side only' accountability technological detail to a more comprehensive representation embracing load consumption profiles and heterogeneous electricity products is therefore necessary.

Table 11. Schematic social accountability with demand load profile disaggregation.

		Uses						
		Q	T&D		Generation		H	GFCF
			tech 1	...	tech n			
Resources	Q	$\overline{P_Q I_{QQ}}$	$P_Q I_{QT\&D}$	$P_Q I_{QG_{tech\ i}}$		$\overline{P_Q Q_H}$	$\overline{P_Q Q_k}$	
	load 1							
	...	$P_{E_{load\ j}}$	$P_{E_{load\ j}}$	$P_{E_{load\ j}}$		$P_{E_{load\ j}}$	-	
	load m	$I_{E_{load\ j}^Q}$	$I_{E_{load\ j}^{T\&D}}$	$I_{E_{load\ j}^{G_{tech\ i}}}$		$E_{H_{load\ j}}$	-	
L	$\overline{wL_Q}$	$wL_{T\&D}$	$wL_{G_{tech\ i}}$					
K	$\overline{rK_Q}$	$rK_{T\&D}$	$rK_{G_{tech\ i}}$					

Source: Own elaboration.

The different load consumption profiles can be represented through a row disaggregation of the previous input-output framework (as show in Table 11). Each new line represents different products, for each corresponding time in which the economy agents consume electricity. The discrete time representation is characterized by different load block levels (load j, j=1,...,m). In turn, the heterogeneous electricity goods at each specific load block present a different combination of production technologies used to provide the less costly system operation. Namely, each load block has its own ratio of technologies used to produce electricity, and base load electricity production can be differentiated from peak units of production. In order to represent this extended arrangement in the same framework, it is necessary to assign different electricity sectors structures to each load level (load jj, jj=1,...,m), as shown in Table 12.

Table 12. Schematic social accountability with heterogeneous electricity production represented.

		Uses						H	GFCF		
		Q	Load 1			...	Load m				
			T&D <sub>1</sub>	Generation <sub>1</sub>		T&D <sub>m</sub>	Generation <sub>m</sub>				
Resources	Q	$\overline{P_Q I_{QQ}}$	$P_Q I_{QT\&D_{jj}}$	$P_Q I_{QG_{load\ jj\ tech\ i}}$	...	...	$\overline{P_Q Q_H}$	$\overline{P_Q Q_k}$			
	load 1										
	...	$P_{E_{load\ j}}$	$P_{E_{load\ j}}$	$P_{E_{load\ j}}$	...	...	$P_{E_{load\ j}}$	-			
	load m	$I_{E_{load\ j}^Q}$	$I_{E_{load\ j}^{T\&D_{load\ jj}}}$	$I_{E_{load\ j}^{G_{load\ jj\ tech\ i}}}$	...	...	$E_{H_{load\ j}}$	-			
L	$\overline{wL_Q}$	$wL_{T\&D_{load\ jj}}$	$wL_{G_{load\ jj\ tech\ i}}$	...	...						
K	$\overline{rK_Q}$	$rK_{T\&D_{load\ jj}}$	$rK_{G_{load\ jj\ tech\ i}}$	...	...						

Source: Own elaboration.

The resulting production structure obtained is capable of addressing simultaneously the previous Sue Wing model capabilities for assessing technological changes and aggregate demand variations and, additionally, qualifies the data framework to applications involving shifting profiles of electricity demand and electricity elasticity

analysis, impossible to attain under the previous approach. Moreover, the new arrangement is also capable to represent correctly the effects on prices of different electricity sector structures, without incurring in the strong simplification of assuming the average electricity price paradigm.

However, the definition of the structure to be applied is only the first step to undertake the data integration. From now on, the paper focuses on defining a consistent procedure to achieve the TD-BU data integration into the extended structure presented in Table 12.

- The first disaggregation problem: Supply-side technology and activities description

The consolidation of economic and engineering data needs to respect a series of initial embedded restrictions belonging to each field: macroeconomic and microeconomic. At the macroeconomic level, the data accountability requires to comply with the principle of double-entry accounting where the total revenue (row) equals the sum of total expenditure (column) for each account, respecting the income balance. This property reflects the constancy of returns to scale and perfect competition assumptions embedded on such accountability benchmark values, originating the initial ‘must follow’ constraints on any TD SAM-based model integration. At the BU microeconomic level, the structure of activities and technologies disaggregation should be respected, and their variable and fixed costs should be adapted to the economic inputs structure (factors and intermediate inputs data).

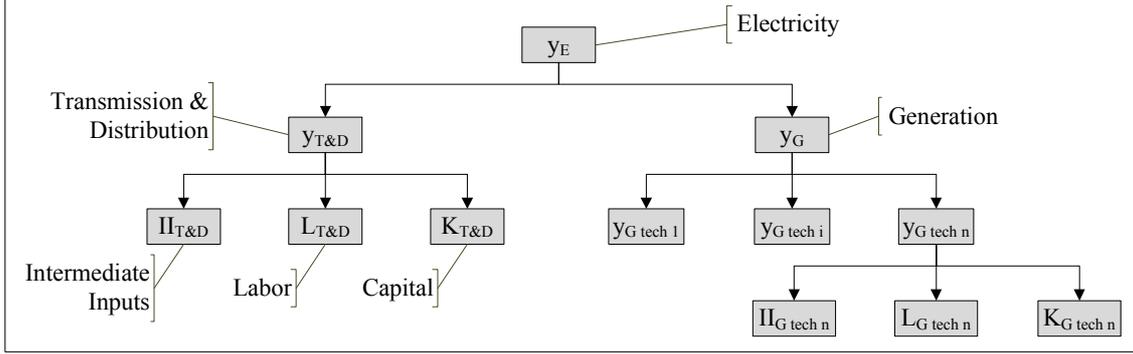
In order to respect the macro-data income constraint, a few restrictions need to be taken into account on the three different disaggregation schemes presented above. Focusing in the scheme presented in Table 10, the electricity production activity is disaggregated on its uses by activities - Transmission & Distribution and Generation – and, additionally, by its generation production technologies (tech  $i$ ,  $i=1, \dots, n$ ). The original output of the electricity sector is given by the sum of its respective column in Table 9 and corresponds to equation II.1. In order to maintain the income balance constraint after the activities disaggregation (column disaggregation), the original resources used in the electricity sector should equal the sum of their new different uses on the new disaggregated activities, as expressed in equation II.2.

$$y_E = \overline{P_Q II_{QE}} + \overline{P_E II_{EE}} + \overline{wL_E} + \overline{rK_E} \quad \text{II.1}$$

$$\begin{bmatrix} \overline{P_Q II_{QE}} \\ \overline{P_E II_{EE}} \\ \overline{wL_E} \\ \overline{rK_E} \end{bmatrix} = \begin{bmatrix} P_Q II_{QT\&D} \\ P_E II_{ET\&D} \\ wL_{T\&D} \\ rK_{T\&D} \end{bmatrix} + \begin{bmatrix} \sum_{tech\ 1}^{tech\ n} P_Q II_{QG_{tech\ i}} \\ \sum_{tech\ 1}^{tech\ n} P_E II_{EG_{tech\ i}} \\ \sum_{tech\ 1}^{tech\ n} wL_{tech\ i} \\ \sum_{tech\ 1}^{tech\ n} rK_{tech\ i} \end{bmatrix} \quad \text{II.2}$$

Once the TD analysis is made, the derivation of microeconomic constraints requires the determination of a specific BU structure. Figure 18 illustrates the arrangement adopted in this paper.

Figure 18. Electricity output disaggregation.



Source: Own elaboration.

The formulation of zero profit and market clearing conditions are necessary in order to maintain the compatibility between the original aggregated electricity sector and the disaggregated alternative. Respecting these conditions avoids the presence of undesirable income leaks in the economy, which would turn the data incompatible with the SAM equilibrium assumptions. Consequently, the electricity sector output must equal the sum of outputs of each activity in which it is disaggregated (equation II.3). Identically, the output of all generation technologies should sum up to the output of the generation activity (equation II.4).

At the very last level of each activity, all outputs are composed by a combination of intermediary inputs and production factors expenses. Therefore, the zero profit condition requires that T&D activity expenses in intermediate inputs ( $P_Q II_{QT\&D}$ ,  $P_E II_{ET\&D}$ ) and production factors ( $wL_{T\&D}$ ,  $rK_{T\&D}$ ) sum up to their respective output level ( $y_{T\&D}$ ), as shown in equation II.5. Likewise, the zero profit condition must also be respected at the generation technology level (equation II.6).

$$y_E = y_{T\&D} + y_G \quad \text{II.3}$$

$$y_G = \sum_{tech\ 1}^{tech\ n} y_{G\ tech\ i} \quad \text{II.4}$$

$$y_{T\&D} = P_Q II_{QT\&D} + P_E II_{ET\&D} + wL_{T\&D} + rK_{T\&D} \quad \text{II.5}$$

$$y_{G\ tech\ i} = P_Q II_{QG\ tech\ i} + P_E II_{EG\ tech\ i} + wL_{G\ tech\ i} + rK_{G\ tech\ i}, \quad \forall i \quad \text{II.6}$$

All together, equations II.1- II.6 correspond to the ‘must follow’ constraints necessary to maintain the SAM initial assumptions. At this point, we still need to determine to more steps of disaggregation: the demand profile and the heterogeneous electricity goods representation.

- The second disaggregation problem: Demand load profile disaggregation

Extending the calibration approach to the time-dependable demand situations faced by consumers follows a similar arrangement. As expressed in Table 11, the heterogeneous consumption feature of the electricity commodity can be dealt with, by transforming the single electricity product into multiple discrete differentiated load block products. The previous operations fulfilled on columns of the accountability matrix to the supply side disaggregation are now replaced by analogous row operations.

Again, in order to respect the income balance constraints, the different uses of electricity (electricity at load block  $j$  used by sector Q, T&D, generation or households) should equal the original amount used of electricity by the sectors as expressed in equations II.7 - II.10.

$$\overline{P_E I_{EQ}} = \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} I_{E_{load\ j}Q} \quad II.7$$

$$P_E I_{ET\&D} = \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} I_{E_{load\ j}T\&D} \quad II.8$$

$$P_E I_{EG_{tech\ i}} = \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} I_{E_{load\ j}G_{tech\ i}}, \quad \forall i \quad II.9$$

$$\overline{P_E E_H} = \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} E_{H_{load\ j}} \quad II.10$$

The intermediate inputs group has to be extended to include the new electricity load level disaggregation. Thus, equations II.5 and II.6 must be replaced by equations II.11 and II.12 respectively:

$$y_{T\&D} = P_Q I_{QT\&D} + \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} I_{E_{load\ j}T\&D} + wL_{T\&D} + rK_{T\&D} \quad II.11$$

$$y_{G_{tech\ i}} = P_Q I_{QG_{tech\ i}} + \sum_{load\ 1}^{load\ m} P_{E_{load\ j}} I_{E_{load\ j}G_{tech\ i}} + wL_{G_{tech\ i}} + rK_{G_{tech\ i}}, \quad \forall i \quad II.12$$

- The third disaggregation problem: Heterogeneous electricity production.

Representing the different production situations faced by the electricity sector at each time-dependable load level follows the same process as describing the generation sector in the form of different production technologies. Nevertheless, the structure presented in the previous Table 10 and Figure 18, was only suitable to represent the situation of only one specific load block, and therefore needs to be extended to include the new heterogeneous production dimension (dimension  $jj$ ).

Therefore, the output of the electricity sector has to include the new electricity load level disaggregation (equation II.13). Additionally, the BU activities disaggregation, intermediate inputs and production factors resources previously represented in equations II.3, II.4, II.11 and II.12 must be applied to each different load level, as represented by equations II.14, II.15, II.16 and II.17 respectively.

$$y_E = \sum_{load\ 1}^{load\ m} y_{E_{load\ jj}} \quad II.13$$

$$y_{E_{load\ jj}} = y_{T\&D_{load\ jj}} + y_{G_{load\ jj}}, \quad \forall jj \quad II.14$$

$$y_{G_{load\ jj}} = \sum_{tech\ 1}^{tech\ n} y_{G_{tech\ i\ load\ jj}}, \quad \forall jj \quad II.15$$

$$y_{T\&D_{load\ jj}} = P_Q I_{QT\&D_{load\ jj}} + \sum_{load\ 1}^{load\ jj=m} P_{E_{load\ j}} I_{E_{load\ j} T\&D_{load\ jj}} +$$

$$+ wL_{T\&D_{load\ jj}} + rK_{T\&D_{load\ jj}}, \quad \forall jj \quad \text{II.16}$$

$$y_{G_{load\ jj\ tech\ i}} = P_Q I_{QG_{load\ jj\ tech\ i}} + \sum_{load\ 1}^{load\ jj=m} P_{E_{load\ j}} I_{E_{load\ j} G_{load\ jj\ tech\ i}} +$$

$$+ wL_{G_{load\ jj\ tech\ i}} + rK_{G_{load\ jj\ tech\ i}}, \quad \forall i, jj \quad \text{II.17}$$

Once more, the income balance constraint must also to be respected after the new column disaggregation in order to maintain the SAM equilibrium. Therefore, equations II.18 - II.25 also need to be taken into account.

$$P_Q I_{QT\&D} = \sum_{load\ 1}^{load\ jj=m} P_Q I_{QT\&D_{load\ jj}} \quad \text{II.18}$$

$$P_{E_{load\ j}} I_{E_{load\ j} T\&D} = \sum_{load\ 1}^{load\ jj=m} P_{E_{load\ j}} I_{E_{load\ j} T\&D_{load\ jj}}, \quad \forall j \quad \text{II.19}$$

$$wL_{T\&D} = \sum_{load\ 1}^{load\ jj=m} wL_{T\&D_{load\ jj}} \quad \text{II.20}$$

$$rK_{T\&D} = \sum_{load\ 1}^{load\ jj=m} rK_{T\&D_{load\ jj}} \quad \text{II.21}$$

$$P_Q I_{QG_{tech\ i}} = \sum_{load\ 1}^{load\ jj=m} P_Q I_{QG_{load\ jj\ tech\ i}}, \quad \forall i \quad \text{II.22}$$

$$P_{E_{load\ j}} I_{E_{load\ j} G_{tech\ i}} = \sum_{load\ 1}^{load\ jj=m} P_{E_{load\ j}} I_{E_{load\ j} G_{load\ jj\ tech\ i}}, \quad \forall i, j \quad \text{II.23}$$

$$wL_{G_{tech\ i}} = \sum_{load\ 1}^{load\ jj=m} wL_{G_{load\ jj\ tech\ i}}, \quad \forall i \quad \text{II.24}$$

$$rK_{G_{tech\ i}} = \sum_{load\ 1}^{load\ jj=m} rK_{G_{load\ jj\ tech\ i}}, \quad \forall i \quad \text{II.25}$$

Finally, in the new disaggregated situation, equations II.1, II.2, II.7 - II.10 and II.13 - II.25 are the new ‘must follow’ constraints necessary to maintain the SAM assumptions and the electricity sector structure. However, these equations do not exhaust all the relevant information contained in BU data. Additional information, not necessarily compatible with the SAM representation must also be respected, as will be shown in the next section for the case of the thermodynamic efficiency of the different technologies used on electricity production.

### III. The thermodynamic efficiency problem

Not all BU information can be translated into use shares of technologies, activities or similar in the SAM framework. As pointed out by Mcfarland (2004), technologies in BU models are typically described as a set of linear activity models based on engineering data of life cycle costs and thermodynamic efficiencies. While the first one is perfectly represented by the process described in the previous section, the thermodynamic properties still need to be addressed in the integration issue. The main point of relevance is to avoid a misleading result in economic models, where the electricity production efficiency could exceed the thermodynamic transformation limit.

“The concept of thermodynamic efficiency for a power plant is well understood in engineering terms as the ratio of the energy content of the electricity produced to the energy content of the fuel input” (Mcfarland, 2004, pag.693). Traditionally, economic formulations make use of production functions capable of combining different inputs (fuel, labor, capital,...) in order to determine the efficient production frontier for a specific final product (in this case represented by the electricity in its different load levels). The layout of these functions, and especially the substitutions elasticities involved, can cause a more than thermodynamically efficient use of a specific fuel in the electricity production, violating the basic laws of thermodynamics.

“For elasticities of substitution” (between fuel and other productive inputs and factors) “greater than zero, price increases of fuels or CO2 relative to capital and labor lead to a greater use of capital and labor to produce a unit of electricity thus improving thermal efficiency” (Mcfarland & Herzog, 2006, pag.641). In fact, this is a main concern in the design of a general equilibrium model. However, even when dealing exclusively with the data adaptation between the TD and BU approaches some limitations occur.

As previously pointed out, the thermodynamic efficiency can be described as the ratio between electricity produced and fuel used by the technology:

$$\eta_{tech\ i} = \frac{q_{electricity\ tech\ i}}{q_{fuel\ tech\ i}} \quad \text{III.1}$$

The TD accountability scheme need to make use of an aggregated equivalent of the diverse products present in the economy, usually representing them as a single output for each sector. In order to achieve this aggregation, the different goods produced are weighted by their respective prices for each specific sector. Consequently, the information contained in such data frameworks represents price-times-quantity dimensions (expenditures or income); meanwhile the engineering thermal efficiency information is provided specifically in quantity terms. For that reason, it is necessary to adjust equation III.1 to the new price relative situation: as expressed by Sue Wing (2008) and Macfarland (2004):

$$\eta_{tech\ i} = \left( \frac{y_{electricity\ tech\ i}}{AP_{electricity}} \right) \bigg/ \left( \frac{y_{fuel\ tech\ i}}{p_{fuel}} \right), \quad \forall i \quad \text{III.2}$$

, where  $\eta_{tech\ i}$  represents the thermodynamic efficiency for tech  $i$ ,  $y_{electricity_{tech\ i}}$  ( $= p_{electricity} * q_{electricity_{tech\ i}}$ ) is the electricity output from tech  $i$ ,  $AP_{electricity}$  is the average price of electricity,  $y_{fuel_{tech\ i}}$  ( $= p_{fuel} * q_{fuel_{tech\ i}}$ ) is the expenditure on fuel by the tech  $i$  and  $p_{fuel}$  is the price of the fuel used.

In summary, the income produced for each technology is transformed in corresponding quantities by dividing their values by the average relative price of the electricity good. However, as stated before in section II, the average electricity prices are not good indicators of the market price behavior in competitive electricity markets.

Due to the marginal character of price determination in these competitive markets, the average electricity price is insufficient to describe the price and cost structure of the electricity sector. Moreover, at marginal competitive markets, the average price would only convey enough information to represent correctly the price behavior of electricity in the exclusive case of the presence of a unique electricity production technology. As a direct consequence, the same problem discussed in the previous section - which was entirely devoted to propose a method to disaggregate the electricity product in corresponding differentiated products related with their different price levels - has also to be addressed to the thermal efficiency equation adaptation.

In the presence of heterogeneous electricity products, dividing the electricity income by the average electricity price does not provide the corresponding relative electricity quantities ( $q_{electricity_{tech\ i}} \neq \frac{p_{electricity_{load\ jj}} q_{electricity_{tech\ i\ load\ jj}}}{AP_{electricity}}$ , where  $p_{electricity_{load\ jj}}$  is the marginal unit cost at load block  $jj$ ), incurring in possible violations of the thermodynamics law. Therefore, it is necessary to consider each load level marginal unit price as the weighting factors for the power plants income in order to correct translate income/expenses into quantity terms, as shown in equation III.3:

$$\eta_{tech\ i\ load\ block\ jj} = \frac{\left( \frac{y_{electricity_{tech\ i\ load\ jj}}}{p_{electricity_{load\ jj}}} \right)}{\left( \frac{y_{fuel_{tech\ i\ load\ jj}}}{p_{fuel}} \right)}, \quad \forall i, jj \quad III.3$$

#### IV. A model of the electricity power sector

The enumeration of variables considered in the section II disaggregation problem, and described by equations II.1, II.2, II.7 - II.10 and II.13 - II.25, gives the following:  $P_Q$ ,  $P_E$ ,  $w$ ,  $r$ ,  $II_{QE}$ ,  $II_{EQ}$ ,  $II_{EE}$ ,  $L_E$ ,  $K_E$ ,  $E_H$ ,  $y_E$ ,  $II_{ET\&D}$ ,  $II_{EG_{tech\ i}}$ ,  $II_{QT\&D}$ ,  $II_{QG_{tech\ i}}$ ,  $L_{T\&D}$ ,  $K_{T\&D}$ ,  $L_{G_{tech\ i}}$ ,  $K_{G_{tech\ i}}$ ,  $II_{E_{load\ j}T\&D}$ ,  $II_{E_{load\ j}G_{tech\ i}}$ ,  $y_{T\&D}$ ,  $y_G$ ,  $y_{G_{tech\ i}}$ ,  $y_{E_{load\ jj}}$ ,  $II_{E_{load\ j}Q}$ ,  $Q_{H_{load\ j}}$ ,  $P_{E_{load\ j}}$ ,  $y_{T\&D_{load\ jj}}$ ,  $y_{G_{load\ jj}}$ ,  $y_{G_{load\ jj\ tech\ i}}$ ,  $II_{QT\&D_{load\ jj}}$ ,  $II_{E_{load\ j}T\&D_{load\ jj}}$ ,  $L_{T\&D_{load\ jj}}$ ,  $K_{T\&D_{load\ jj}}$ ,  $II_{QG_{load\ jj\ tech\ i}}$ ,  $II_{E_{load\ j}G_{load\ jj\ tech\ i}}$ ,  $L_{G_{load\ jj\ tech\ i}}$  and  $K_{G_{load\ jj\ tech\ i}}$ .

Analyzing them at the benchmark levels, the products and factors prices can be fixed at given levels allowing working only with relative corresponding quantities. Therefore, the prices  $\overline{P_Q}$ ,  $\overline{P_E}$ ,  $\overline{w}$ ,  $\overline{r}$  can be considered as fixed parameters, as also the quantities

$\overline{II_{QE}}, \overline{II_{EQ}}, \overline{II_{EE}}, \overline{L_E}, \overline{K_E}$  and  $\overline{E_H}$  can be taken directly from the SAM information and from the relative prices assumed. Following equation II.1, the output from the electricity sector ( $\overline{y_E}$ ) can be also considered as a parameter.

Variables  $II_{ET\&D}, II_{EG_{tech\ i}}, II_{QT\&D}, II_{QG_{tech\ i}}, L_{T\&D}, K_{T\&D}, L_{G_{tech\ i}}, K_{G_{tech\ i}}, II_{E_{load\ j}T\&D}, II_{E_{load\ j}G_{tech\ i}}, y_{T\&D}, y_G$  and  $y_{G_{tech\ i}}$  are intermediary auxiliary variables obtainable directly from other variables through the equations described. Furthermore, knowing the demand profile of the other sectors (Q) and households (H) in the benchmark year, it is also possible to determine  $II_{E_{load\ j}Q}$  and  $Q_{H_{load\ j}}$  (equations IV.1 and IV.2).

$$II_{E_{load\ j}Q} = (Q \text{ load profile in } j)\overline{II_{EQ}}, \quad \forall j \quad \text{IV.1}$$

$$Q_{H_{load\ j}} = (H \text{ load profile in } j)\overline{Q_H}, \quad \forall j \quad \text{IV.2}$$

At this point, we still need to determine the electricity marginal price at each load block ( $P_{E_{load\ j}}$ ), which should be a known result at the benchmark year. In practice, the participation shares of activities and technologies ( $y_{T\&D_{load\ jj}}$  and  $y_{G_{load\ jj\ tech\ i}}$ ) can be also acquired in advance from the specific electricity sector operation at the benchmark.

Finally, it is in the production factors participations ( $L_{T\&D_{load\ jj}}, K_{T\&D_{load\ jj}}, L_{G_{load\ jj\ tech\ i}}, K_{G_{load\ jj\ tech\ i}}$ ) and intermediate inputs proportions ( $II_{QT\&D_{load\ jj}}, II_{E_{load\ j}T\&D_{load\ jj}}, II_{QG_{load\ jj\ tech\ i}}, II_{E_{load\ j}G_{load\ jj\ tech\ i}}$ ) that the incompatibility between TD and BU data takes place. In order to deal with this problem, firstly we need to represent the expenditure variables related to these participations as shares of the total expenditure made in the electricity sector, as shown below (equations IV.3 and IV.6).

$$\begin{aligned} y_{E_{load\ jj}} &= \left( s_{y_E}^{y_{E_{load\ jj}}} \right) y_E, \quad \forall jj \\ y_{T\&D_{load\ jj}} &= \left( s_{y_{E_{load\ jj}}}^{y_{T\&D_{load\ jj}}} \right) y_{E_{load\ jj}}, \quad \forall jj \\ y_{G_{load\ jj}} &= \left( s_{y_{E_{load\ jj}}}^{y_{G_{load\ jj}}} \right) y_{E_{load\ jj}}, \quad \forall jj \\ \left( s_{y_{E_{load\ jj}}}^{y_{T\&D_{load\ jj}}} \right) + \left( s_{y_{E_{load\ jj}}}^{y_{G_{load\ jj}}} \right) &= 1, \quad \forall jj \end{aligned} \quad \text{IV.3}$$

$$\begin{aligned} y_{G_{load\ jj\ tech\ i}} &= \left( s_{y_{G_{load\ jj}}}^{y_{G_{load\ jj\ tech\ i}}} \right) y_{G_{load\ jj}}, \quad \forall i, jj \\ \sum_{tech\ 1}^{tech\ n} \left( s_{y_{E_{load\ jj}}}^{y_{G_{load\ jj\ tech\ i}}} \right) &= 1, \quad \forall jj \end{aligned} \quad \text{IV.4}$$

$$\begin{aligned} \bar{P}_Q II_{QT\&D_{load\ jj}} &= \left( s_{y_{T\&D_{load\ jj}}}^{II_{QT\&D_{load\ jj}}} \right) y_{T\&D_{load\ jj}}, \quad \forall jj \\ \bar{P}_{E_{load\ j}} II_{E_{load\ j}T\&D_{load\ jj}} &= \left( s_{y_{T\&D_{load\ jj}}}^{II_{E_{load\ j}T\&D_{load\ jj}}} \right) y_{T\&D_{load\ jj}}, \quad \forall j, jj \\ \bar{W} L_{T\&D_{load\ jj}} &= \left( s_{y_{T\&D_{load\ jj}}}^{L_{T\&D_{load\ jj}}} \right) y_{T\&D_{load\ jj}}, \quad \forall jj \\ \bar{r} K_{T\&D_{load\ jj}} &= \left( s_{y_{T\&D_{load\ jj}}}^{K_{T\&D_{load\ jj}}} \right) y_{T\&D_{load\ jj}}, \quad \forall jj \end{aligned} \quad \text{IV.5}$$

$$\left( s_{y_{T\&D_{load\ jj}}}^{II_{QT\&D_{load\ jj}}} \right) + \sum_{load\ j=1}^{load\ j=m} \left( s_{y_{T\&D_{load\ jj}}}^{II_{E_{load\ j}T\&D_{load\ jj}}} \right) + \left( s_{y_{T\&D_{load\ jj}}}^{L_{T\&D_{load\ jj}}} \right) + \left( s_{y_{T\&D_{load\ jj}}}^{K_{T\&D_{load\ jj}}} \right) = 1, \quad \forall jj$$

$$\begin{aligned}
\bar{P}_Q II_{QG_{load\ jj\ tech\ i}} &= \left( s_{yG_{load\ jj\ tech\ i}}^{II_{QG_{load\ jj\ tech\ i}}} \right) y_{G_{load\ jj\ tech\ i}}, \quad \forall i, jj \\
\bar{P}_{E_{load\ j}} II_{E_{load\ j} G_{load\ jj\ tech\ i}} &= \left( s_{yG_{load\ jj\ tech\ i}}^{II_{E_{load\ j} G_{load\ jj\ tech\ i}}} \right) y_{G_{load\ jj\ tech\ i}}, \quad \forall i, j, jj \\
\bar{w} L_{G_{load\ jj\ tech\ i}} &= \left( s_{yG_{load\ jj\ tech\ i}}^{L_{G_{load\ jj\ tech\ i}}} \right) y_{G_{load\ jj\ tech\ i}}, \quad \forall i, jj \\
\bar{r} K_{G_{load\ jj\ tech\ i}} &= \left( s_{yG_{load\ jj\ tech\ i}}^{K_{G_{load\ jj\ tech\ i}}} \right) y_{G_{load\ jj\ tech\ i}}, \quad \forall i, jj \\
\left( s_{yG_{load\ jj\ tech\ i}}^{II_{QG_{load\ jj\ tech\ i}}} \right) &+ \sum_{load\ j=1}^{load\ j=m} \left( s_{yG_{load\ jj\ tech\ i}}^{II_{E_{load\ j} G_{load\ jj\ tech\ i}}} \right) + \\
&+ \left( s_{yG_{load\ jj\ tech\ i}}^{L_{G_{load\ jj\ tech\ i}}} \right) + \left( s_{yG_{load\ jj\ tech\ i}}^{K_{G_{load\ jj\ tech\ i}}} \right) = 1, \quad \forall i, jj
\end{aligned}
\tag{IV.6}$$

It would be a trivial process to transform engineering costs information into production factors representation, and consequently acquire all above share values, under a perfectly compatible accountability approach. However, in the ‘real world’, the different costs structure, data sources (company accountability vs. technical characteristics) and data availability difficult this process.

The trick used by Sue wing (2008), and also by this paper, to achieve a suitable process to turn compatible the engineering and economic costs representation is to allow a certain degree of freedom at the previously represented shares within the electricity sector, while minimizing their deviations with the micro and macroeconomic data. Assuming that the activities shares related with BU data can be known in advance at the benchmark year, it is possible to calibrate the economic data according to the engineering information, and by so, determine the intermediate ( $s_{y\dots}^{II_{\dots}}$ ) and factors shares ( $s_{y\dots}^L$  and  $s_{y\dots}^K$ ) for each technology (i) and load block (jj), while at the same time respecting a previously determined fitness measure of how well the calibrated costs activities measures fits the real activities shares in the benchmark year.

Thus, the process translates into a simple minimization problem, aiming to achieve the lowest possible upper ( $d_U$ ) and lower ( $d_L$ ) deviations of the benchmarked BU shares and thermal efficiency estimated, while respecting the ‘must follow’ constraints to maintain the SAM equilibrium assumptions:

$$\begin{aligned}
\text{Min.: } & \left\{ \sum_{jj} \left( d_U^{s_{yE} y_{Eload jj}} + d_U^{y_{Eload jj}} \right) + \sum_{jj} \left( d_U^{T\&Dload jj} + d_L^{T\&Dload jj} \right) \right. \\
& + \sum_{jj} \left( d_U^{s_{yEload jj} y_{Gload jj}} + d_L^{s_{yEload jj} y_{Gload jj}} \right) + \sum_{jj} \sum_i \left( d_U^{s_{yEload jj} y_{Gload jj} tech i} + d_L^{s_{yEload jj} y_{Gload jj} tech i} \right) \\
& + \sum_{jj} \left( d_U^{s_{T\&Dload jj} \Pi_{QT\&Dload jj}} + d_L^{s_{T\&Dload jj} \Pi_{QT\&Dload jj}} \right) \\
& + \sum_j \sum_{jj} \left( d_U^{s_{T\&Dload jj} \Pi_{Eload j T\&Dload jj}} + d_L^{s_{T\&Dload jj} \Pi_{Eload j T\&Dload jj}} \right) \\
& + \sum_{jj} \left( d_U^{s_{yT\&Dload jj} L_{T\&Dload jj}} + d_L^{s_{yT\&Dload jj} L_{T\&Dload jj}} \right) + \sum_{jj} \left( d_U^{s_{yT\&Dload jj} K_{T\&Dload jj}} + d_L^{s_{yT\&Dload jj} K_{T\&Dload jj}} \right) \\
& + \sum_{jj} \sum_i \left( d_U^{s_{yGload jj} \Pi_{QGload jj} tech i} + d_L^{s_{yGload jj} \Pi_{QGload jj} tech i} \right) \\
& + \sum_j \sum_{jj} \sum_i \left( d_U^{s_{yGload jj} \Pi_{Eload j Gload jj} tech i} + d_L^{s_{yGload jj} \Pi_{Eload j Gload jj} tech i} \right) \\
& + \sum_{jj} \sum_i \left( d_U^{s_{yGload jj} L_{Gload jj} tech i} + d_L^{s_{yGload jj} L_{Gload jj} tech i} \right) \\
& + \sum_{jj} \sum_i \left( d_U^{s_{yGload jj} K_{Gload jj} tech i} + d_L^{s_{yGload jj} K_{Gload jj} tech i} \right) \\
& \left. + \sum_{jj} \sum_i \left( d_U^{\eta_{tech i} load block jj} + d_L^{\eta_{tech i} load block jj} \right) \right\}
\end{aligned}$$

Subject to the must follow constraints represented by equations II.1, II.2, II.7 - II.10, II.13 - II.25, IV.1 and IV.2 and to the following restrictions:

$$\begin{aligned}
y_{Eload jj} - \bar{s}_{yE}^{y_{Eload jj}} y_E + d_U^{s_{yE} y_{Eload jj}} - d_L^{s_{yE} y_{Eload jj}} &= 0, \quad \forall jj \\
y_{T\&Dload jj} - \bar{s}_{yEload jj}^{T\&Dload jj} y_{Eload jj} + d_U^{s_{yEload jj} T\&Dload jj} - d_L^{s_{yEload jj} T\&Dload jj} &= 0, \quad \forall jj \\
y_{Gload jj} - \bar{s}_{yEload jj}^{y_{Gload jj}} y_{Eload jj} + d_U^{s_{yEload jj} y_{Gload jj}} - d_L^{s_{yEload jj} y_{Gload jj}} &= 0, \quad \forall jj \\
y_{Gload jj} tech i - \bar{s}_{yGload jj}^{y_{Gload jj} tech i} y_{Gload jj} + d_U^{s_{yEload jj} y_{Gload jj} tech i} - d_L^{s_{yEload jj} y_{Gload jj} tech i} &= 0, \quad \forall i, jj \\
\bar{P}_Q \Pi_{QT\&Dload jj} - \bar{s}_{T\&Dload jj}^{\Pi_{QT\&Dload jj}} y_{T\&Dload jj} + d_U^{s_{T\&Dload jj} \Pi_{QT\&Dload jj}} - d_L^{s_{T\&Dload jj} \Pi_{QT\&Dload jj}} &= 0, \quad \forall jj
\end{aligned}$$

$$\bar{P}_{Eload j} II_{Eload j T\&Dload jj} - \bar{s}_{T\&Dload jj}^{II_{Eload j T\&Dload jj}} y_{T\&Dload jj} + d_U^{II_{Eload j T\&Dload jj}} - d_L^{II_{Eload j T\&Dload jj}} = 0, \quad \forall j, jj$$

$$\bar{W}_{L_{T\&Dload jj}} - \bar{s}_{y_{T\&Dload jj}}^{L_{T\&Dload jj}} y_{T\&Dload jj} + d_U^{L_{T\&Dload jj}} - d_L^{L_{T\&Dload jj}} = 0, \quad \forall jj$$

$$\bar{K}_{T\&Dload jj} - \bar{s}_{y_{T\&Dload jj}}^{K_{T\&Dload jj}} y_{T\&Dload jj} + d_U^{K_{T\&Dload jj}} - d_L^{K_{T\&Dload jj}} = 0, \quad \forall jj$$

$$\bar{P}_Q II_{QGload jj tech i} - \bar{s}_{y_{Gload jj tech i}}^{II_{QGload jj tech i}} y_{Gload jj tech i} + d_U^{II_{QGload jj tech i}} - d_L^{II_{QGload jj tech i}} = 0, \quad \forall i, jj$$

$$\bar{P}_{Eload j} II_{Eload j Gload jj tech i} - \bar{s}_{y_{Gload jj tech i}}^{II_{Eload j Gload jj tech i}} y_{Gload jj tech i} + d_U^{II_{Eload j Gload jj tech i}} - d_L^{II_{Eload j Gload jj tech i}} = 0, \quad \forall i, j, jj$$

$$\bar{W}_{L_{Gload jj tech i}} - \bar{s}_{y_{Gload jj tech i}}^{L_{Gload jj tech i}} y_{Gload jj tech i} + d_U^{L_{Gload jj tech i}} - d_L^{L_{Gload jj tech i}} = 0, \quad \forall i, jj$$

$$\bar{K}_{Gload jj tech i} - \bar{s}_{y_{Gload jj tech i}}^{K_{Gload jj tech i}} y_{Gload jj tech i} + d_U^{K_{Gload jj tech i}} - d_L^{K_{Gload jj tech i}} = 0, \quad \forall i, jj$$

$$\frac{y_{Etech i load jj}}{\bar{P}_{Eload jj}} - \bar{\eta}_{tech i load block jj} \frac{y_{Ifuel tech i load jj}}{\bar{P}_{Ifuel}} + d_U^{\eta_{tech i load block jj}} + d_L^{\eta_{tech i load block jj}} = 0, \quad \forall i, jj$$

In the presence of perfectly compatible data between BU and TD models, all deviations would present null values and the BU data would perfectly fit in the TD framework. In the real world, this equality never happens and the deviations values provide direct measures of the quantity in which each share deviates from the original BU data.

Additionally, the use of goal programming approach to solve this problem clearly presents advantages in comparison to Sue Wing (2008) non-linear approach, reflected by the linear formulation of the multi-criteria decision adopted in comparison to the quadratic alternative. However, as the model is formulated in this paper, any deviation variable presents the same magnitude of repercussion on the objective function. This allows the possibility to occur a concentration of deviations in a specific load block or technology. In order to introduce dispersion in the deviation results, obtaining the smaller deviations at each activity and load block according their incremental error percentage, an extension is made necessary and the deviations variables should be pondered by their specific magnitudes.

## V. Data

The main topic of this paper addresses the availability and compatibility of different data frameworks. As could be foreseen, in the real world neither the availability nor the

compatibility of data are easy issues to deal with, and therefore, it was necessary to make a certain amount of assumptions regarding the data requirements of such disaggregation.

As previously described, the process described in this paper is illustrated with data from the Spanish economy. The macroeconomic data was acquired from the 2005 symmetric input-output table assembled by the Spanish National Institute of Statistics (INE – “Instituto Nacional de Estadística”). The 73 industries partition provided by the national accountability was integrated into seven representative sectors, according to their relationship with the electricity sector. Inside these, two groups can be outlined: the fuel suppliers (Carbon, Oil/Nuclear and Gas) and the typical electricity demanders (Manufactures, Transport and Services) shown on Table 13.

Table 13. Symmetric Input-Output table for Spain in the year 2005.

	SECTORS							INSTITUTIONS			CAPITAL		FOREIGN RELATIONS	
	Manufactures	Coal	Oil/Nuclear	Gas	Electricity	Transport	Other Services	Households	Non-profit institutions	Government	GFCF	Inventory changes	UE exports	RW exports
Manufactures	396912	243	18084	4975	1966	6568	97410	117770	0	7033	201876	635	101043	37225
Coal	378	0	1	4	1887	5	74	34	0	0	0	14	4	1
Oil/Nuclear	6214	30	5533	29	3072	6243	3093	7724	0	0	0	219	3677	3806
Gas	2011	0	4	1	3146	120	1078	1281	0	0	0	1	111	0
Electricity	8963	88	73	29	5410	741	10358	6095	0	0	0	10	353	65
Transport	24847	31	746	6	271	20639	14985	12054	0	1587	217	0	11024	5070
Other Services	92784	100	1381	258	4773	12276	179882	346331	8047	154626	42358	4	24679	10755
Labor	102997	301	391	198	1270	13025	216237	0	0	0	0	0	0	0
Capital	101730	60	3227	2115	9099	16683	246069	0	0	0	0	0	0	0
Social Contributions	31093	96	130	71	505	3892	60626	0	0	0	0	0	0	0
Production Taxes	-1116	-17	77	41	438	-114	4652	0	0	0	0	0	0	0
Product Tax	149	19	625	27	-154	2791	14087	54474	0	494	22592	0	0	-89
UE imports	140098	58	4237	0	479	6530	20945	0	0	0	0	0	0	0
RW imports	84677	1391	5132	0	23	2078	8757	0	0	0	0	0	0	0

Source: Own elaboration based on data from the National Institute of Statistics (INE - Spain).

Unity: Millions of Euros.

In turn, the microeconomic data requires a more extensive explanation. First of all, the electricity commodity was differentiated by its consumption time. Five load blocks were used to represent these different electricity commodities, three representing the demand levels on working days (low, medium and high demand), and two representing the holidays (low and high demand) (Table 14).

Table 14. Load block information.

	Duration (hours in the year)		
Low Load Working Day	2000	Days	Hours
Medium Load Working Day	3000	Working Days	6000
High Load Working Day	1000	Holiday Days	2760
Low Load Holiday Day	1840	Total	8760
High Load Holiday Day	920		
Total	8760		

Source: Own elaboration based on year 2005. Unity: described in the table.

In order to maintain simplicity, electricity taxes, imports and exports were excluded from the disaggregation scheme and considered exogenously given at the levels provided by the Spanish National Institute of Statistics Input-Output table.

Two hypotheses were assumed to derive the electricity demand profile of each agent<sup>23</sup>. The fuel producers (Coal, Oil/Nuclear and Gas) and the manufacturing sector are assumed to be interruptible electricity demanders and as assumed by the “Atlas de la Demanda Eléctrica Española” (Indel, REE, 1997) sustain a linear, flatter, consumption profile. The remaining agents (Transport, Other Services and Households) have their consumption profile at each load block defined by the residual hourly system profile behavior.

Theoretical reference prices were taken for each product and production factor. As the TD representation is described in expenditure terms, there is no problem in assuming any absolute price level at the benchmark year because any change on this value is nothing more than a change in the unit that measures the quantity of products and factors. However, the same does not apply to the electricity sector prices. Traditionally, BU data are described in specific quantity terms. The disaggregation of the sector into load blocks proposed by this paper requires weighting the BU quantities by their load block prices in order to achieve the correct participation shares on the new TD extended approach. Marginal prices for each load block were estimated from information obtained from the Spanish Market Operator (OMEL).

Data from the year 2007 was adapted as reference to the bottom-up information due the lack of statistics relative to 2005 Spanish system operation.

The expenditure held at each load block electricity production was estimated from its proportional produced energy, taken from “Red Eléctrica” electricity operation database (REE, ESIOS, 2007), and the load block relative prices (see Table 15). Simultaneously, the participation share of each generation technology at each load block production can be also estimated from the same source (Table 16).

<sup>23</sup> The agents referred correspond to all macroeconomic sectors, excluding electricity, and the households.

Table 15. Share of electricity expenses for each load block electricity production.

	Low Load Working Day	Medium Load Working Day	High Load Working Day	Low Load Holiday Day	High Load Holiday Day	Total
Electricity expenses share by load block	20.20%	37.93%	14.20%	17.18%	10.50%	100.00%

Source: Own elaboration, based on “Red Eléctrica” (REE), ESIOS, 2007. Unity: percentage. Share:

$$\bar{s}_{yE}^{yEload jj}$$

Table 16. Generation technology participation on each electricity load block production.

	Low Load Working Day	Medium Load Working Day	High Load Working Day	Low Load Holiday Day	High Load Holiday Day
Nuclear	21.65%	17.41%	16.57%	23.40%	20.14%
Coal	26.45%	24.07%	22.72%	27.21%	25.18%
Fuel-Gas	1.64%	1.39%	1.64%	1.47%	1.15%
Combined Cycle	23.26%	26.65%	29.88%	18.27%	22.95%
Hydro	5.49%	10.23%	9.71%	7.60%	10.21%
Pumping	0.07%	1.06%	1.83%	0.17%	1.34%
Wind	10.78%	8.87%	7.95%	10.87%	9.56%
Others Special Regime	10.66%	10.33%	9.70%	11.00%	9.47%
Total	100.00%	100.00%	100.00%	100.00%	100.00%

Source: Own elaboration, based on “Red Eléctrica” (REE), ESIOS, 2007. Unity: percentage. Share:

$$\bar{s}_{yGload jj}^{yGload jj tech i}$$

T&D and generation shares on electricity expenditures are assumed for simplicity as constant through each load block level, and are estimated from the 2005 tariff payments (Real decreto 2392/2004) and from the statistic report of the Spanish Electrical Industry Association (UNESA, 2005) ( $\bar{s}_{yEload jj}^{T\&Dload jj}=33.65\%$  and  $\bar{s}_{yEload jj}^{yGload jj}=66.35\%$ ).

Spanish power plants thermal efficiency in combustibles transformation is described in Table 17.

Table 17. Inverse thermal efficiency in combustible transformation by Spanish thermal power plants.

	Nuclear	Coal	Fuel-Gas	Combined Cycle
Coal		2.6		
Oil-Nuclear	3.15			
Gas			2.5	1.99

Source: Own elaboration. Unity: p.u. Share:  $\bar{\eta}_{tech i}$ .

Finally, the bottom-up technological description in terms of intermediate inputs, load block electricity demand and production factors are shown in Table 18. It was assumed that all fuel inputs were used by electricity generation companies (T&D share for this inputs are considered null). Participation shares for Fuel spending on each generation technology were calculated assuming the Spanish electricity expenses and production. The Labor/Capital expenditure proportion on each technology follows EIA data and Sue Wing (2008) assumptions. It was also assumed that the intermediate electricity use in generation is originated from pumping consumption only, and its distribution through load blocks is taken from REE (2007) pumping consumption data.

Capital and labor shares in T&D expenditure were calculated based on data from the Spanish Electrical Industry Association data (UNESA, 2005). The T&D intermediate inputs shares were calculated from the residual expenditure after the previously described generation expenditures, except in the already mentioned case of fuel sectors (considered null as describe above). The distribution of electricity intermediate demand through each load block (represented mainly by losses) was assumed linear in relation to the energy production in the corresponding load block.

Table 18. Electricity bottom-up description in terms of intermediate inputs and factors participation.

	T&D	Generation							
		Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others Special Regime
Manufactures	2.72%								
Coal			31.68%						
Oil-Nuclear		26.73%							
Gas				80.81%	69.00%				
Electricity									
Low Load Working Day	1.41%						56.23%		
Medium Load Working Day	2.78%						2.99%		
High Load Working Day	1.06%						0.65%		
Low Load Holiday Day	1.23%						36.09%		
High Load Holiday Day	0.78%						4.03%		
Transport	0.37%								
Other Services	16.99%								
Labor	21.87%	12.87%	14.85%	7.07%	8.00%	30.00%		21.25%	22.78%
Capital	50.77%	60.40%	53.47%	12.12%	23.00%	70.00%		78.75%	77.22%
Total	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Source: Own elaboration. Unity: percentage. Shares:  $(\bar{s}_{T\&Dload\ jj}^{IIQT\&Dload\ jj}, \bar{s}_{T\&Dload\ jj}^{IIEload\ jT\&Dload\ jj}, \bar{s}_{yT\&Dload\ jj}^{LT\&Dload\ jj}, \bar{s}_{yT\&Dload\ jj}^{KT\&Dload\ jj}, \bar{s}_{yGload\ jj\ tech\ i}^{IIQGload\ jj\ tech\ i}, \bar{s}_{yGload\ jj\ tech\ i}^{IIEload\ jGload\ jj\ tech\ i}, \bar{s}_{yGload\ jj\ tech\ i}^{LGload\ jj\ tech\ i}, \bar{s}_{yGload\ jj\ tech\ i}^{KGload\ jj\ tech\ i})$ .

## VI. Results

The model solution provides a disaggregated version of the input-output table presented in Table 13 (see Annex I – Extended Input-Output table). Firstly, five new rows are included in the matrix as result of the disaggregation of the electricity commodity into five differentiated products, each one corresponding to a different load demand level. However, it is in the matrix columns that the larger changes occur.

The electricity sector is disaggregate into five different sectors, each one with a specific structure in order to represent the different generation and transmission mix necessary to provide diverse demanded levels. Inside each of these new sub-sectors, the T&D and generation activities are dismembered accordingly their expenditure composition. In total, the single electricity sector column is disaggregated into 45 new columns (five

groups of nine activities, eight originated from generation technologies and one from the T&D activity).

The results obtained by the model are capable to perfectly adjust the technologies participation at each load block to the benchmark year results by adjusting the T&D expenses and the use of intermediate inputs and productive factors at each activity. Consequently, the deviations on electricity expenses by load block and by activity are null at the final estimated values (the calibrated values reflect exactly the proportions described in table Table 16).

Also, the expenses in T&D and generation represent 10,396.43 and 20,496.87 millions of euros (33.65% and 66.35% of the total electricity expenses respectively).

The incompatibility present in TD and BU data structures are therefore transmitted to the intermediate inputs and production factors deviations. Table 19 summarizes the technological deviations of the model estimated values compared to the benchmark BU shares presented in Table 18.

Table 19. Intermediate inputs and productions factors activities deviations from benchmark shares.

	T&D	Generation							
		Nuclear	Coal	Fuel-Gas	Combine d Cycle	Hydro	Pumping	Wind	Others Special Regime
Manufactures	16.05% (489.86%)								
Coal			40.69% (28.43%)						
Oil-Nuclear		85.62% (220.23%)							
Gas				40.00% (-50,50%)	65.88% (-4.53%)				
Electricity	L L	36.91%					56.23%		
	W D	(2511,2%)					(0.00%)		
	M L	2.78%					2.99%		
	W D	(0.00%)					(0.00%)		
	H L	1.06%					0.65%		
	W D	(0.00%)					(0.00%)		
	L L	1.23%					36.09%		
	H D	(0.00%)					(0.00%)		
Transport	2.21% (489.86%)								
Other Services	38.97% (129.33%)								
Labor	0.00% (-100%)	0.00% (-100%)	3.15% (-78.80%)	0.00% (-100%)	0.00% (-100%)	27.77% (-7.43%)		16.03% (-24.56%)	19.43% (-14.70%)
Capital	0.00% (-100%)	14.38% (-76.19%)	56.16% (5.04%)	60.00% (395.00%)	34.12% (48.37%)	72.23% (3.19%)		83.97% (6.63%)	80.57% (4.34%)
Total	100% (Not applicable)								

Source: Own elaboration. Unity: percentage. Model estimated shares are presented firstly and the percentage variation in relation to the benchmark data (Table 18) are presented under brackets.

As can be seen the deviations obtained are not depreciable. Evaluating by activity, only the pumping generation incur on a perfect match. Non thermal generation technologies (hydro, wind and other technologies from special regime) present a medium degree of deviations (from 3% to 25% from the benchmark values).

Nevertheless, is in the thermal generation technologies that the higher issues occurs. As can be seen, Nuclear, Coal-fuel and Combined Cycle technologies present very different figures to expenditure shares on combustibles when comparing BU and the model results to TD data. The motive for such deviations is the presence of a trade-off between maintaining the BU thermal efficiency values or approximate the intermediate fuel expenditure present in the TD data. The mismatch between these values causes repercussions through the factors shares at these technologies, and consequently, affects the labor and capital available to other activities.

The estimations to the thermal efficiency at the TD extended approach can be seen in Table 20. The lower ratio of fuel use by thermal technologies satisfactorily respects the thermal fuel efficiency constraints presented in Table 17, however as explained above this occurs in detriment of fuel expenditure ratio deviations described in Table 19.

Table 20. Thermal fuel efficiency by load block.

	Nuclear	Coal	Fuel-Gas	Combined Cycle
Coal		2.49 (-4.17%)		
Oil-Nuclear	1.12 (-64.34%)			
Gas			2.5 (0.00%)	1.57 (-21.19%)

Source: Own elaboration. Unity: p.u. and percentage. Model values presented firstly and followed by percentage variation in relation to Table 17 in parenthesis.

As a result, all errors occurred in the electricity generation estimations also propagate to the T&D activity and are summed up to its own deviations. The intermediate T&D expenditure on manufactures, transports and other services acts to equilibrate the misbalance on fuel expenses from the thermal units acquiring elevated deviation figures. The same happens in the production factors level, where the previously benchmarked stocks of capital and labor pertaining to the T&D activity migrates entirely to the generation activities.

## VII. Conclusions

This paper introduces a Top-down detailed procedure addressing the integration of not only the technological production richness of bottom-up data, but also of a detailed load block electricity demand description into a social accountability framework. The electricity demand disaggregation is important for electricity policy evaluations because even while two electrons could be essentially represented as a homogeneous product, the impracticability of storing electricity causes the electricity at each specific time to act like a time-dependable heterogeneous commodity. Particularly, peak and off-peak production structures differ significantly and have a meaningful weight in diverse electricity policy evaluations.

Zero profit and market clearing conditions embodied in the TD data structures were derivative to the new extended Input-output representation of the economy. The deviations on the Top-down data originated from the inclusion of bottom-up technologically detail and load block information were minimized through goal programming. The linear formulation of the multi-criteria decision problem adopted clearly present advantages when compared to previous non-linear examples found in similar literature.

Spanish electricity market data was used to illustrate the calibration process. Specific intermediate input shares and economic productive factors were calculated for the country specific case. Nonetheless, two issues limited the relative success of the calibration process when applied to the Spanish case.

First of all, the calibration process presented on this paper does not discriminate the deviations contributions on the objective function. Therefore, concentrating all deviations in a specific load block or distributing them through all load blocks can represent multiple feasible solutions, as can be seen in Table 19 where all electricity consumption deviations for the T&D activity are concentrated in one load block. This clearly represents an unwanted solution that should be addressed in the calibration problem.

Secondly, and more significant, the calibration procedure clearly pointed out to a disproportionate disparity between the share of combustibles use on BU and TD data estimations. The discrepancy between the estimated combustibles weights in thermal generation expenses caused repercussions on the electricity generation factors deviations and, consequently, in the T&D factors and intermediate input shares.

The detailed procedure presented in this paper to integrate technological information and, by the first time in the literature, load block differentiation on demand and production levels can have a qualitative and quantitative effect on TD applications. The framework proposed improve the electricity sector representation and widen the application TD model like Computable General Equilibrium models to previously problematic issues under this approach. Representing the time dependability of electricity production enable such models to analyze different electricity elasticity behaviors (mostly important in issues like the formulation of demand response programs) or tariff design problems where assembling correctly energy-only and access-tariffs require time discrimination. In particular, the correct representation of peaking demand should be taken as crucial in E3 models because they involve considerable economical, environmental and technical inefficiencies due their very low utilization factors.

Additional studies are being developed to deal with the two issues identified on the calibration process for the Spanish data. An alternative deviation weight is been implemented to avoid the concentration of errors in specific decision variables, meanwhile the global calibration result is maintained. By the other side, improvements in data quality is been pursued, to avoid the use of different years data sources and improve activities, combustibles and load block share estimations.

Likewise, a series of additional research can be pointed out as refinements of the research described in this paper. First of all, improvements can be made on the

estimations of the technological shares used by each activity, which in most cases for this paper are assumed constant whatever the load block. Specially, the T&D activity was represented in these paper equations as load block dependable, but the data used assumed equal ratios between different load blocks. Represent correctly the load block expenditures is crucial to address accordingly the access-tariff time differentiations. Moreover, an evident extension to the model presented would incorporate additional disaggregations to commercial and operational activities assumed in this paper contained within the T&D activity.

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## Annex I – Extended Input-Output table

		SECTORS													
		Manufactures Coal Oil/Nuclear Gas T&D					Electricity								
							Low Load Working Day								
Manufactures	Coal	Oil/Nuclear	Gas	T&D	Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others	Special Regime		
Products	Manufactures	396912	243	18084	4975	57								295	
	Coal	378	0	1	4		421	16							
	Oil/Nuclear	6214	30	5533	29	347				227		95		146	
	Gas	2011	0	4	1	112		27	484						
	Low Load Working Day	2046	20	17	7	30					506				
	Medium Load Working Day	3069	30	25	10	58					27				
	High Load Working Day	1023	10	8	3	22					6				
	Low Load Holiday Day	1883	19	15	6	26					325				
	High Load Holiday Day	941	9	8	3	16					36				
	Transport	24847	31	746	6	8	88								
Other Services	92784	100	1381	258	357		16	261							
Productive Factors	Labor	102997	301	391	198										
	Capital	101730	60	3227	2115	1066	585	8	218			352			
Taxes	Social Contributions	31093	96	130	71									505	
	Production Taxes	-1116	-17	77	41									438	
	Product Tax	149	19	625	27									-154	
Foreign Relations	UE imports	140098	58	4237	0									479	
	RW imports	84677	1391	5132										23	

		Sectors Electricity																			
		Medium Load Working Day										High Load Working Day									
		T&D	Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others Special Regime	T&D	Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others Special Regime		
Products	Manufactures						254				88				368						
	Coal								283				1297						105		
	Oil/Nuclear							19	437							21	323				
	Gas				6						143		7								
	Low Load Working Day	58			41						16			50							
	Medium Load Working Day	13			16						3			19							
	High Load Working Day	696			18						193			22							
	Low Load Holiday Day	78			12						22			14							
	High Load Holiday Day																				
	Transport				251			29	233				303		77	25	172	162			
	Other Services																				
Productive Factors	Labor		543		750		354		200						512	6	148				
	Capital																				
Taxes	Social Contributions																				
	Production Taxes																				
	Product Tax																				
Foreign Relations	UE imports																				
	RW imports																				

		Medium Load Holiday Day										High Load Holiday Day							
		Sectors Electricity																	
		T&D	Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others Special Regime	T&D	Nuclear	Coal	Fuel-Gas	Combined Cycle	Hydro	Pumping	Wind	Others Special Regime
Products	Manufactures	49									30								
	Coal			368								208							
	Oil/Nuclear	1297					105		81		239							206	46
	Gas				21	323							10	248					
	Electricity						467			15						260			
		25						25		30						14			
	50						5		12						3				
	19						300		13						167				
	22						33		9						19				
	14																		
	Transport	7								4									
	Other Services	303		77	25	172	162		88	185		44	12	246	220				
Productive Factors	Labor																		
	Capital			512	6	148			301	299	554	290	3					157	
Taxes	Social Contributions																		
	Production Taxes																		
	Product Tax																		
Foreign Relations	UE imports																		
	RW imports																		

		Sectors		Institutions			Capital		Foreign relations	
		Transport	Other Services	Households	Non-profit institutions	Government	GFCF	Inventory changes	UE exports	RW exports
Products	Manufactures	6568	97410	117770		7033	201876	635	101043	37225
	Coal	5	74	34				14	4	1
	Oil/Nuclear	6243	3093	7724				219	3677	3806
	Gas	120	1078	1281				1	111	0
	Low Load Working Day	105	1466	863						
	Medium Load Working Day	68	943	555						
	High Load Working Day	217	3036	1787				10	353	65
	Low Load Holiday Day	115	1603	943						
	High Load Holiday Day	237	3309	1947						
	Transport	20639	14985	12054		1587	217		11024	5070
Other Services	12276	179882	346331	8047	154626	42358	4	24679	10755	
Productive Factors	Labor	13025	216237							
	Capital	16683	246069							
Taxes	Social Contributions	3892	60626							
	Production Taxes	-114	4652							
	Product Tax	2791	14087	54474		494	22592			-89
Foreign Relations	UE imports	6530	20945							
	RW imports	2078	8757							